Can Stablecoins be Stable?[*]

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Abstract

This paper provides a general framework for analyzing the stability of stable-coins, cryptocurrencies pegged to a traditional currency. We study the problem of a monopolist platform that can earn seigniorage revenues from issuing stablecoins. We characterize stablecoin issuance-redemption and pegging dynamics under various degrees of commitment to policies. Even under full commitment, the stablecoin peg is vulnerable to large demand shocks. Backing stablecoins with collateral helps to stabilize the platform but is costly for the platform's equity (token) holders. Combined with collateral, decentralization can act as a substitute for commitment.

Keywords: Stablecoins, Cryptocurrencies, Target Leverage, Dynamic Games, Coase Conjecture

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1 Introduction

A stablecoin is a cryptocurrency designed to maintain a peg with an official currency. Stablecoins can allegedly cater to investors' demand for alternative means of payments by combining the benefits of blockchain technology with the stability of traditional currencies. These cryptocurrencies have recently gained in popularity as the market value of stablecoins grew from \$3 billion in 2019 to \$152 billion in June 2022.[1] Confronted with this rapid development, along with multiple depegging events and crashes,[2] legislators started introducing new initiatives to regulate stablecoins.[3]

This paper proposes a framework to study the stability of various pegging mechanisms and the optimal design of stablecoin platforms. Our analysis speaks to a wide range of protocols: algorithmic supply adjustments (e.g., Terra), partial collateralization (e.g., Frax), and decentralization of the issuance process (e.g., DAI). Our paper focuses on the incentive problems faced by stablecoin issuers and analyzes the contribution of collateral and decentralization to the stability of stablecoins.

In our model, a monopolist platform caters to a time-varying demand for stablecoins. Users, who value price stability, enjoy liquidity benefits from owning stablecoins when their price is pegged to some unit of account. These liquidity benefits depend on the total stock of stablecoins, which can reflect liquidity satiation or network effects. As a monopolistic issuer, the platform can extract seigniorage revenues from these liquidity benefits, similarly to a (private or central) bank. Like a bank that can overprint money, a stablecoin platform has a tendency to overissue stablecoins, which ultimately undermines the peg. In the face of this time-inconsistency problem, the main technological proposition of stablecoins is the possibility to rely on smart contracts to enforce commitment to specific policies.

Our dynamic model has two building blocks. First, the monopolistic stablecoin platform chooses its issuance-repurchase policy of stablecoins and its interest-rate policy, paid in stablecoins to users. The platform may also collateralize stablecoin issuance with a safe asset. In this case, the platform must hold as colleral a fraction of the par value of outstanding stablecoins. To finance these policies, the platform can freely issue equity

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[1]https://www.statista.com/statistics/1255835/stablecoin-market-capitalization/
[2]For example, the collapse of the Terra-Luna platform in May 2022. See Appendix A.
[3]For instance, the US Congress is working on a STABLE (Stablecoin Tethering and Bank Licensing Enforcement) Act; in the UK, the Treasury has launched the "UK regulatory approach to cryptoassets and stablecoins: Consultation and call for evidence".

shares, called equity tokens, to external investors. Second, users price the stablecoin competitively, given the liquidity benefits they expect to derive from owning stablecoins and the interest payments from the platform. We characterize the equilibrium price of stablecoins and platform's equity tokens and provide conditions under which the peg holds.

First, we study a stablecoin platform that can fully commit to all of its policies through immutable smart contracts. We show that there always exists an equilibrium in which stablecoins and equity tokens are worth zero. This equilibrium arises because both stablecoin dividends—liquidity benefits and interest payments—depend themselves on the value of stablecoins. Without an external anchor, stablecoins may have no value even with full commitment, similarly to fiat money.

In this full-commitment benchmark, a second equilibrium exists in which the platform maintains the peg unless it is hit by a large negative demand shock. In this equilibrium, equity tokens have positive value and represent a claim to the platform's future seigniorage revenues. To maintain a stable price, the platform reacts to a negative demand shock by repurchasing (issuing) stablecoins to reduce (increase) the supply. In an expansion phase, the platform generates revenues by minting new stablecoins. In a contraction phase, the platform finances stablecoin buybacks by issuing additional equity shares and diluting legacy token holders.[4] After a large negative demand shock, however, future cash flows from seigniorage revenues may be so low that the platform cannot finance the necessary stablecoin repurchase to maintain the peg, even with a complete dilution of equity. The price then falls below par to reflect the imbalance between supply and demand and equity token are worth zero. This prediction is consistent with the collapse of the fully algorithmic platform Terra-Luna in 2022. Following a run on the platform—a large negative demand shock in our model—the protocol reacted by minting increasing quantities of Luna equity token to burn/buy back Terra stablecoins and the peg broke as Luna's price fell to zero.[5]

Our analysis under full commitment provides two additional insights. First, we obtain necessary existence conditions for an uncollateralized platform: Demand for stablecoin must grow over time so that new issuance gains cover the costs equity token holders face to defend the peg. This result resonates with informal claims that stablecoin protocols

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[4]This ability of stablecoin platforms to issue equity shares or "tokens" continuously at no substantial cost is crucial to allow pure algorithmic stablecoins to perform the equivalent of open market operations without holding any tangible asset on their balance sheets.

[5]In Appendix A, we provide a descriptive analysis of the May 2022 stress for the five largest stablecoin platforms.

are Ponzi schemes to the extent that they rely on user demand growth. Second, collateral fosters the platform stability: A fully collateralized platform is akin to a a narrow bank and can always maintain the peg as it finances stablecoin repurchases entirely with collateral. A stablecoin issuer, however, can earn profit only if the liquidity benefits enjoyed by stablecoin users exceed the cost of holding collateral. In other words, a stablecoin fully backed by Treasuries must command a higher convenience yield than government debt. In general, the optimal collateralization ratio trades off stability benefits with the cost of locking collateral.

We then analyze the stability of a stablecoin scheme under a weaker form of commitment. In practice, a stablecoin protocol retains some discretion over key parts of its algorithm to preserve adaptability to new market developments and technical issues. We thus relax our assumption that all policies can be fully programmed via smart contracts. While the platform can still commit to an interest rate policy, it now chooses its issuance-repurchase policy sequentially. The platform then faces a durable-good monopolist problem as in Coase (1972): When issuing new stablecoins, it does not take into account the negative impact of this issuance on existing stablecoin users whose liquidity benefits depend on the total stock. This feature generates overissuance relative to the commitment solution.

Our main result in this case is that a programmable state-contingent interest rule can ensure stability even without commitment to issuance and repurchase. If the interest rate the platform pays to users is constant, the leverage ratchet effect of Admati, DeMarzo, Hellwig, and Pfleiderer (2018) and DeMarzo and He (2021) applies to our setting: The platform never finds it optimal to buy back stablecoins. In this case, the platform can neither maintain the peg nor capture seigniorage revenues. Such an equilibrium can be sustained, however, with an interest rate policy that penalizes overissuance and incentivizes the platform to conduct repurchases. This design can achieve local stability even for an uncollateralized platform that faces no direct cost from interest payments in stablecoins.

Lastly, we consider a stablecoin platform that decentralizes issuance and redemption of its stablecoin. DAI is a prominent example of decentralized platforms, which anyone with access to the Ethereum platform is able to mint freely. These so-called vault owners can issue stablecoins by locking the required fraction of collateral. The platform then earns an income flow from charging a fee to decentralized issuers (vaults). We find that decentralization can substitute for a commitment technology. With decentralization, vault owners arbitrage price deviations from the peg. This new arbitrage condition ties the platform's hands when it comes to setting stablecoin interest rates and vault fees. In

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addition, decentralization transforms the platform's income from issuance into a flow of income. We show that these features solve the platform's time-inconsistency problem and incentivize equity token owners to implement the full-commitment issuance policy, similarly to the leasing solution to Coase's (1972) problem.

Related literature Our paper contributes to an emerging literature on stablecoins. Several works provide a description of stablecoin protocols and pegging mechanisms (Arner, Auer, and Frost, 2020; Berentsen and Schär, 2019; Bullmann, Klemm, and Pinna, 2019; ECB, 2019; Eichengreen, 2019; G30, 2020). Gorton et al. (2022) estimate the convenience yield on stablecoins and argue that technological advances and reputation formation can make stablecoins money-like. In contrast, we take the existence of users' liquidity benefit as given and study stability issues of different protocols. Lyons and Viswanath-Natraj (2020) argue that arbitrage by vault owners is a key stabilizing force, a finding consistent with our analysis of decentralized protocols. In addition, we propose a general model to analyze the incentive problems of the equity owners of the stablecoin platform. Kereiakes, Kwon, DiMaggio, and Platias (2019) and DiMaggio and Platias (2019) propose partial equilibrium models specific to the Terra-Luna stablecoin. While we also find that a stablecoin peg should be robust to small shocks, we stress that all stablecoin protocols exhibit fragility to large negative demand shocks unless issuance is fully backed by collateral. Uhlig (2022) proposes a model to uncover the specific mechanisms of the Luna crash.[6]

In closely related contemporaneous work, Li and Mayer (2022) study stablecoin peg dynamics by considering a reserve management problem for a centralized stablecoin platform. Our result that large negative shocks can lead to depegging echoes their finding. Unlike them, we analyze the platform's time-consistency problem and consider both undercollateralized centralized protocols and decentralized protocols. Thus, our model can speak to a wider range of existing stablecoin designs.[7]

As it strives to defend a peg, a stablecoin platform faces a problem similar to a central

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[6]In computer sciences, Klages-Mundt and Minca (2019, 2020) develop models that feature endogenous stablecoin price and exogenous collateral and find deleveraging spirals and liquidation in a system with imperfectly elastic stablecoin demand. Gudgeon, Perez, Harz, Livshits, and Gervais (2020) simulate a stress-test scenario for a DeFi protocol and find that excessive outstanding debt and the drying up of liquidity can cause the lending protocol to become undercollateralized.

[7]Their framework also differs from ours in important ways: their model includes restrictions to equity issuance, shocks to reserves (akin to collateral in our model) rather than to stablecoin demand, and stablecoins that mature instantaneously.

bank under a fixed exchange rate regime (Obstfeld, 1996). Routledge and Zetlin-Jones (2021) build on this analogy to model undercollateralized currencies. They show that commitment to a dynamically adjusting exchange rate policy can fend off self-fulfilling runs, similar to suspension of deposit convertibility in the model by Diamond and Dybvig (1983). While we also highlight the role of programmable rules (smart contracts) in defending the peg, our model focuses on the incentives of the platform to overissue stablecoins. In addition, our framework allows us to analyze fully algorithmic (unbacked) stablecoins and equity holders' incentives to recapitalize the platform following large negative shocks.

In studying stabilization mechanisms across stablecoin types and the failure of governance incentives to recapitalize undercollateralized systems, our paper is connected to the corporate finance literature that examines firm shareholders' attitudes toward leverage. In work by Black and Scholes (1973) and Myers (1977), firm shareholders do not have incentives to voluntarily reduce leverage, because this always implies a transfer of wealth to existing creditors. Admati, DeMarzo, Hellwig, and Pfleiderer (2018) generalize these findings to multiple asset classes of debt and with agency frictions and document a *leverage ratchet effect*, whereby shareholders never have any incentive to delever. In a continous-time framework, DeMarzo and He (2021) show that a firm cannot capture any tax benefit of debt if issuance is unrestricted, due to this ratchet effect. Our framework and techniques are related: Their firm corresponds to our stablecoin platform and the tax benefits of debt to the liqudity benefits for stablecoin users. Our contribution is to consider various commitment technologies (collateral, smart contracts, decentralization) that restore the platform's ability to earn seigniorage revenues. In particular, smart contracts can be related to debt convenants, as studied by Smith and Warner (1979); Bolton and Scharfstein (1990); Aghion and Bolton (1992); and Donaldson, Piacentino, and Gromb (2020). Relatedly, Malenko and Tsoy (2020) show that threat to credibility can discipline equity holders to repurchase debt.

Our work also relates to the financial implications of Coase's (1972) conjecture that a monopolist producing durable goods competes against its future self and does not capture any markup. As noted by Calvo (1978), this commitment problem applies to monetary authorities that can earn seigniorage revenues, like the stablecoin platform in our model. We show that the decentralization stablecoin model transforms upfront seigniorage revenues into a flow of payments, and thus preserves a platform's monopolist market power, similar

to the leasing solution to Coase's (1972) conjecture.[8]. Relatedly, Goldstein et al. (2022) show that issuing utility tokens dilutes the maket power of a monopolist platform by transforming a flow of services into a durable good.

More broadly, our paper contributes to the literature that applies finance theory to model token adoption, and valuations (e.g. Cong, Li, and Wang, 2020a,b; Hinzen, John, and Saleh, 2022) and to a fast-growing literature on central bank digital currencies (e.g. Ahnert, Hoffmann, and Monnet, 2022; Benigno, Schilling, and Uhlig, 2022; Brunnermeier, James, and Landau, 2021; Fernandez-Villaverde, Sanches, Schilling, and Uhlig, 2021).[9]

2 General Environment

In this section, we describe our model of stablecoins. The central premise of our analysis is that users enjoy utility benefits from holding stablecoins issued by the platform, as they would from money or bank deposits. Our model also embeds users' preferences for stable means of payment. As a result, the stablecoin platform can generate seigniorage revenues if (but only if) it can maintain a peg between the stablecoin price and some target unit of account. We describe the formal building blocks of the model below.

2.1 Stablecoin Demand

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space that satisfies the usual conditions. All agents are risk neutral with an exogenous discount rate of $r > 0$.[10] Time is continuous with $t \in [0, \infty)$.

We consider a platform that issues stablecoins. Stablecoins are a liability of the platform that trade at (endogenous) price p_t expressed in the unit of account. The outstanding stock of these stablecoins at time t is C_t. Stablecoins have value because users enjoy direct utility from holding them: At time t, holding stablecoins generates utility flow $U(A_t, p_t C_t)$ per unit, with A_t, an exogenous driver of stablecoin utility value. To fix ideas, one could interpret variable A_t as the value of some unmodeled cryptoassets, which proxies for users'

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[8]In a corporate finance setting, Hu, Veras, and Ying (2022) points out that rolling over short-term debt is akin to the leasing solution. In our model, however, as in practice, stablecoins are long-lived assets so we do not consider instantaneous maturity as a design choice.

[9]The literature on cryptocurrencies that are not meant to be used as money is now too long to do it justice here.

[10]Alternatively, we can interpret the model as written under a fixed risk-neutral measure that is independent of the stablecoin platform's policies.

demand for alternative means of payment. The utility derived from holding stablecoins can be thought of as a liquidity benefit users enjoy because stablecoins are a form of money.[11] We denote the marginal utility benefit from holding an additional unit of stablecoin, or its *convenience yield*, as $\ell(A_t, p_t C_t) p_t \equiv \partial U(A_t, p_t C_t) / \partial C_t$. In what follows, we impose some restrictions on the properties of ℓ, which is a sufficient statistics for our analysis.

Assumption 1. *The convenience yield on stablecoins $\ell(A, pC)$ is (i) positive and continuously differentiable in both arguments; (ii) bounded with $0 \leq \ell(A, pC) \leq r$; (iii) homogeneous of degree 0; and (iv) equal to 0 if the stablecoin price p is not pegged to 1. Finally, (v) the product of the convenience yield and the total value of stablecoins $\ell(A, pC) pC$ is single-peaked with $\lim_{x \to \infty} \ell(A, x) x = 0$.*

Property (i) rules out negative marginal utility from stablecoin holdings and ensures differentiability. Properties (ii) and (iii) are technical assumptions that ensure, respectively, that the stablecoin price is well defined and that the problem ultimately economizes on one state variable. Property (iv) states that stablecoin owners enjoy a liquidity benefit only if it is pegged to the unit of account. This assumption is meant to capture, in a tractable way, the fact that users value the stablecoin as a means of payment to the extent that its issuer can maintain its parity with the unit of account.[12] The peg at 1 is chosen for convenience and because it corresponds to market practice, but our results do not depend upon it; only the real value of stablecoin holdings pC matters. Finally, Property (v) ensures that the optimal amount of stablecoins is interior. A class of functions that satisfy Assumption 1 is $\ell(A, C) = \kappa \exp((A/C)^{-\alpha} - (A/C)^{-\beta})$ for $\beta > \alpha > 0$ and $\kappa = r / \max_a \{\ell(a, 1)\}$. In this example, when C is low, the convenience yield is increasing in C, as more stablecoins are in circulation, but eventually declines as the stock becomes too large.[13]

The variable A_t that drives stablecoins' demand has the following law of motion:

$$dA_t = \mu A_t dt + \sigma A_t dZ_t + A_{t^-}(S_t - 1) dN_t, \tag{1}$$

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[11] Our reduced-form specification can be microfounded, assuming that stablecoins are essential in order to conduct some transactions.

[12] Without this "extreme-peg" assumption, a stablecoin could have value, even though there is no active management of the supply of stablecoins to stabilize its price, as a standard cryptocurrency. This assumption allows us to precisely characterize the optimal equilibrium policies in Lemma 2 and Proposition 3.

[13] This reduced-form specification captures both network effects from stablecoin adoption and the fact that a large supply of stablecoin satiates users' liquidity needs.

where dZ_t is the increment of a standard Brownian motion and dN_t is a Poisson process with constant intensity $\lambda > 0$ adapted to $\mathcal{F}$. The size of a downward jump, $-\ln(S)$, is exponentially distributed with parameter $\xi > 0$ and the expected jump size is $\mathbb{E}[S - 1] = -1/(\xi+1)$. The Poisson process generates large negative shocks to stablecoin demand that can be thought of as news about the usefulness of the stablecoin or speculative attacks. Overall, the expected growth rate of stablecoin demand is given by $\mu - \lambda/(\xi + 1)$, which we assume is strictly lower than the discount rate r.

Finally, there exists a safe asset that the platform can hold as collateral to back the issuance of stablecoins. This collateral trades in a competitive market at price p_t^k with

$$dp_t^k = \mu^k p_t^k dt. \tag{2}$$

This specification implies that collateral delivers a (safe) return μ^k with $\mu^k < r$. The difference between the discount rate and the return on collateral, $r - \mu^k$, can be interpreted as a convenience yield enjoyed by collateral asset owners. For our analysis, this feature generates a cost from holding collateral for the stablecoin platform.[14] Examples of this asset include cash, government securities, or bank deposits denominated in the target currency. In Figure 1, we provide a sketch of a balance sheet for a generic centralized platform.

2.2 Platform Operation

We will analyze both a centralized and a decentralized platform. For clarity, we postpone the description of a decentralized platform to Section 5.

Definition 1 (Centralized Platform Policies). *A sequence of policies for the platform is an issuance-repurchase policy $\{d\mathcal{G}_t\}_{t\geq 0}$; an interest rate policy $\{\delta_t\}_{t\geq 0}$ paid in stablecoins, with $\delta_t \geq 0$; a collateral purchase policy $\{d\mathcal{M}_t\}_{t\geq 0}$; and a stochastic default time τ.*

The main policy of the platform is the issuance(-repurchase) policy $\{d\mathcal{G}_t\}_{t\geq 0}$. A positive (negative) value of $d\mathcal{G}_t$ corresponds to an issuance (repurchase) of stablecoins at price p_t at date t. The platform can also pay interest to stablecoin owners. As in practice, this

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[14]Our assumption of a safe collateral asset comes with some loss of generality because some stablecoin platforms are implicitly or explicitly backed by cryptoassets. In this case, the collateral price would likely be correlated with demand process A_t. It is intuitive, however, that such correlation would reduce the usefulness of collateral as a hedge against demand fluctuations. From a technical standpoint, introducing correlation would significantly complicate the analysis.

Platform

Collateral	Stablecoins
Future Seigniorage	Equity Tokens

Figure 1: Sketch of a Centralized Platform Balance Sheet.

interest is paid in stablecoins, not in the unit of account. A platform may hold collateral, with $d\mathcal{M}_t$ denoting the change in collateral value held by the platform at date t.

There exists a useful analogy between the stablecoin platform and a central bank. When it issues stablecoins ($d\mathcal{G}_t > 0$), the platform receives a payment $p_t d\mathcal{G}_t$ from users in the unit of account. Similarly, when it credits the account of a depository institution with reserves, the central bank receives an asset in exchange. The stablecoin's interest policy, whereby every stablecoin user is credited with $\delta_t \geq 0$ units of stablecoins per unit owned, is akin to an interest payment on reserves. Finally, collateral holdings of the platform correspond to a central bank's holdings of foreign reserves.

In line with most actual stablecoin designs such as DAI, we assume the platform maintains a constant ratio between its collateral holdings and the stock of stablecoins.

Assumption 2. *The platform maintains a fixed collateralization ratio $\varphi \in [0, 1]$, that is,*

$$K_t = \varphi C_t. \tag{3}$$

Assumption 2 simplifies our analysis, in that it eliminates collateral as a state variable. It can be shown that the platform would implement such a fixed ratio if it were subject to a minimum collateralization ratio $K_t \geq \varphi C_t$.[15] The uncollateralized case $\varphi = 0$ corresponds

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[15]Holding a buffer of collateral above the minimum requirement would be suboptimal in equilibrium

to a so-called "pure algorithmic stablecoin" whereas the fully-collateralized one $\varphi = 1$ is typically referred to as a "narrow stablecoin" in reference to narrow banks. Our model captures such heterogeneity and can speak to the optimal collateral ratio for a platform.

Laws of Motion   The platform's policies imply the following laws of motion for the stock of stablecoin outstanding, C_t, and the value of its collateral, denoted K_t:

$$dC_t = \delta_t C_t dt + d\mathcal{G}_t, \tag{4}$$

$$dK_t = \mu^k K_t dt + d\mathcal{M}_t. \tag{5}$$

Equation (4) is the law of motion for stablecoins. The first term on the right-hand side captures the contribution of interest payment policy δ_t to stablecoin issuance. It must be treated separately from the active issuance component $d\mathcal{G}_t$, because the interest policy increases the stablecoin stock without compensation for the platform. Equation (5) is the law of motion for the collateral value. The first term on the right-hand side corresponds to passive changes in collateral value. The second term corresponds instead to active changes in value due to purchases or sales. Collateral policy $d\mathcal{M}_t$ is fully determined by issuance policy $d\mathcal{G}_t$ and interest policy δ_t at date t, because $dK_t = \varphi dC_t$ under Assumption 2.[16]

Jump Notation   As demand is subject to both Brownian shocks to and jumps in the value of cryptoassets in our model, we allow the platform's policies to also feature jumps. A jump represents a discrete, instantaneous change in a variable. We denote the value of a variable X just before and after the jump by X_{t^-} and X_t, respectively.[17]

2.3   Stablecoin Pricing and the Platform's Objective

Stablecoin Pricing   Users price the stablecoin competitively, taking as given the platform's policies. They enjoy two income streams from holding stablecoins: the direct utility benefits when the price is pegged and interest payments, with respective value $\ell_t p_t$ and $\delta_t p_t$ per unit of stablecoin. Should the platform default, an instantaneous liquidation procedure

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because collateral is costly ($\mu^k < r$). For the same reason, the restriction to a collateralization ratio between 0 and 1 comes without loss generality.

[16] Law of motion (5) can alternatively be written $dK_t = S_t^k dp_t^k + p_t^k dS_t^k$, with S_t^k the quantity of collateral held by the platform. The term $d\mathcal{M}_t$ in (5) corresponds to $p_t^k dS_t^k$.

[17] X_{t^-} denotes the left limit $X_{t^-} = \lim_{h \to 0} X_{t-h}$.

applies in which stablecoin owners are treated as pari-passu creditors. They receive any platform's collateral up to the parity value of stablecoins. At date t, the competitive stablecoin price, given the platform's continuation policies, is thus

$$p_t = \mathbb{E}_t \left[\int_t^\tau e^{-r(s-t)} \left(\ell_s + \delta_s \right) p_s ds + e^{-r(\tau-t)} \varphi \right]. \tag{6}$$

Users compute expected future cash flows by forming rational expectations over the platform's policies from date t onward. Upon liquidation of the platform, users receive φ per stablecoin, as the collateralization ratio is lower than the par value of stablecoins ($\varphi \leq 1$).

Platform's Objective The platform starts with no stablecoin outstanding at date 0; that is, $C_{0^-} = 0$, and maximizes its value E_0, which is the sum of the issuance benefits net of collateral purchases.

$$E_0 = \max_{\varphi, \tau, \{\delta_t, d\mathcal{G}_t\}_{t \geq 0}} \mathbb{E}_0 \left[\int_0^\tau e^{-rt} \left(p_t d\mathcal{G}_t - d\mathcal{M}_t \right) \right], \tag{7}$$

where price p_t is given by equation (6). When the platform is liquidated, as $\varphi \leq 1$, all its collateral is used to partially repay stablecoin users. Hence, the platform receives zero payoff upon liquidation. As a monopolistic issuer, the platform has price impact. For instance, if it issues a large amount of stablecoins, the price may drop and the platform would then earn the post-repurchase price on the total issuance.

As we will see, a platform's ability to implement at future dates a policy chosen at date 0 depends on its commitment power. A central technological proposition of stablecoins is that rules and procedures can be programmed in advance through algorithms—so-called *smart contracts*. In many cases, however, platforms retain some flexibility over parts of the algorithm for technical maintenance, future adaptability, or to decrease vulnerability to hacking. To capture these concerns and to reflect heterogeneity in smart contracts' credibility and transparency, we analyze optimal policies under varying degrees of commitment.

Our last assumption rules out Ponzi schemes by the stablecoin platform.

Assumption 3. *The equilibrium policies must satisfy the no-Ponzi-game condition:*

$$\lim_{T \to \infty} \mathbb{E}_t[e^{-rT} p_T C_T] = 0 \quad \forall t \geq 0.$$

Assumption 3 states that the platform cannot sustain the value of stablecoins by simply issuing new stablecoins. In other words, the value of a platform must rely on the creation of liquidity benefits for the owners of stablecoins, and not on the overaccumulation of debt.[18]

2.4 Discussion of the Environment

Platform Fees We assume that the platform interest rate is never negative, that is, $\delta_t \geq 0$. Doing so simplifies the analysis, as only (costly) buy-backs can be used to reduce stablecoin supply.[19] This assumption also corresponds to the practice of the main stablecoin platforms: Terra notoriously subsidized platform usage by paying an annual interest rate of 20%; DAI's interest rate typically fluctuates between 1% and 7%; and Tether does not pay any interest or levy fees.

Peg vs. Redemption Rights In our model, the platform does not provide redemption rights to investors. Instead, investors must trade in a competitive market to exchange their stablecoins for the unit of account, and the platform must administer the peg through supply adjustments. To the extent the platform maintains the peg, however, investors are effectively guaranteed a fixed exchange rate between stablecoins and the unit of account. Not defending the peg is observationally equivalent to ceasing to redeem stablecoins at par.

Platform Competition We focus on the analysis of an economy that features a single stablecoin platform. In practice, several stablecoin platforms compete to cater to users' demand for alternative means of payment. Although we refrain from modeling competition and the entry of platforms for parsimony, we can interpret the platform's convenience yield as investors' residual demand for a platform's stablecoins after accounting for supply from other platforms. The only requirement is that the platform enjoys some market power, which would arise naturally with payment network effects as in Cong, Li, and Wang (2020a).

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[18]Although our analysis excludes Ponzi games in a technical sense, the stablecoin equilibria studied in the following sections have features that are casually associated with Ponzi schemes such as requiring a positive growth rate of demand for stablecoins.

[19]In theory, negative interest rates would provide the platform with an additional tool to reduce the supply of stablecoin by effectively taxing or diluting legacy stablecoin holders. Doing so would allow the platform to maintain a nominal peg, but stablecoin users would face a similar loss as in a depegging event.

Equity Tokens As in most traditional corporate finance settings, it is only the total value of equity (or market capitalization) that matters for equilibrium, and thus we do not need to separately keep track of the nominal quantity of equity tokens outstanding and their price per unit. Consequently, we also abstract from the exact mechanism that the platform uses to buy back stablecoins by diluting governance token holders.

3 Full Commitment

In this section, we analyze the problem of a stablecoin platform that can commit to all future policies. This theoretical full-commitment benchmark corresponds to a platform with immutable smart contracts that govern all policies in all contingencies, including the issuance and repurchase of stablecoins.[20] This benchmark provides minimal necessary conditions for a stablecoin platform to have positive value and be able to maintain parity.

For this analysis, the only constraint on the platform's policy choices at date 0 is that its equity cannot become negative at some future date t—that is, limited liability applies. To clearly highlight the role of this constraint, we first consider a benchmark with unlimited liability in Section 3.1 and then introduce limited liability in Section 3.2.

3.1 Unlimited Liability Benchmark

In this unlimited liability benchmark, the platform's equity value may become negative. In this setting, there is no default so we set $\tau = \infty$. The platform chooses a stablecoin issuance-redemption policy $\{d\mathcal{G}_t\}_{t\geq 0}$, an interest policy $\{\delta_t\}_{t\geq 0}$, and a collateralization rate φ to maximize the value of the platform at date 0 given by

$$E_0 = \max_{\varphi, \{\delta_t, d\mathcal{G}_t\}_{t\geq 0}} \mathbb{E}\left[\int_0^\infty e^{-rt}\left(p_t d\mathcal{G}_t - d\mathcal{M}_t\right) \ A_0, C_{0^-} = 0\right] \tag{8}$$

subject to (6), (4), and, (3). The platform's value is the net present value of issuance proceeds net of collateral purchase costs. The program is solved given: the law of motion for stablecoins (4) implied by the issuance policy and the initial condition $C_{0^-} = 0$; the collateralization rule (3); and the competitive pricing function for stablecoins (6) at any

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[20]Routledge and Zetlin-Jones (2021) demonstrates how smart contracts can implement and provide commitment to the platform's policies.

date t, given policies chosen by the platform for dates $s \geq t$.

Our first result is that even under full commitment, there exists an equilibrium with zero stablecoin price, no stablecoin issuance and zero platform value.

Proposition 1. *There always exists a zero-price equilibrium in which $p_t = 0$, for all $t \geq 0$.*

The zero-price equilibrium arises because there is no anchor between the stablecoin and the unit of account as the interest is paid in stablecoins. To see why a zero-price equilibrium exists, suppose users enjoy no liquidity benefit, which arises when $p > 1$. The sole dividend from the stablecoin is then the interest rate, δp. Hence, the only solution to the pricing equation, $rp = \delta p$, is $p = 0$ under the no-Ponzi condition (Assumption 3). In this case, the platform has no value and zero issuance is optimal if stablecoins are collateralized $(\varphi > 0)$.[21]

Proposition 1 shows that stablecoins, like any fiat money, are fragile: Stablecoins may be worth zero even when issuance and repurchase are fully programmable and implementable. Having shown this result, we now consider equilibria with positive stablecoin value, if any. Under full commitment and with unlimited liability, there exists an equilibrium in which the stablecoin has value and the platform enjoys seigniorage revenues.

Proposition 2. *With full commitment and unlimited liability, the equilibrium with positive stablecoin price features a target demand ratio $A_t/C_t = A_t/C_{ul}^\star(A_t) = a_{ul}^\star$ for all t with*

$$C_{ul}^\star(A) = \arg \max_C \{\ell(A, C)C . \tag{9}$$

The interest rate policy at demand ratio a_{ul}^* is $\delta_{ul}^\star = r - \ell(a_{ul}^\star)$ to peg the stablecoin price to 1 and is not determined otherwise. The platform sets collateralization ratio $\varphi_{ul}^\star = 0$.

As we show formally in the Appendix, the platform value is the present value of liquidity benefits enjoyed by investors net of the collateral holding costs,

$$E_0 = \mathbb{E}\left[\int_0^\infty e^{-rt}\left(\ell(A_t, C_t)C_t + (\mu^k - r)\varphi C_t\right)dt \, A_0, C_{0^-} = 0\right] \tag{10}$$

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[21]To be complete, there exists another equilibrium in which the platform does not capture liquidity benefits but with $p = \varphi$. In this equilibrium, the platform issues stablecoins and immediately defaults, which links the stablecoin price to the collateral backing it. The platform value, however, is still zero.

with $\ell(A,C)C$ the instantaneous total seigniorage revenues. This equivalence is intuitive, because the platform captures all gains from trade. Maximizing the platform value E_0 with unlimited liability thus becomes a static optimization problem to the extent the platform can maintain the peg. In this case, the optimal collateralization rate is $\varphi_{ul}^\star = 0$ because holding collateral is costly. Given current demand A, an interior optimum $C_{ul}^\star(A)$ for stablecoin supply exists under Assumption 1. Homogeneity of the liquidity benefit, $\ell(A,C)$, further implies that $C_{ul}^\star(A)$ is linear in A, and we call $a_{ul}^\star$ the *target demand ratio*.

The need to maintain the peg, $p_t = 1$, in order to capture liquidity benefits determines the platform's interest policy. In equilibrium, the demand ratio a_t is constant, so we only need to specify $\delta_{ul}^\star \equiv \delta(a_{ul}^\star)$. It is easy to verify that the peg holds when $\delta_{ul}^\star$ is given, as in Proposition 2, because for all t we then have

$$p_t = \frac{\ell(a_{ul}^\star) + \delta_{ul}^\star}{r} = 1. \tag{11}$$

Proposition 2 implies that the platform issues (buys back) stablecoins when demand A_t increases (decreases) in order to implement its target demand ratio. This policy reflects the supply adjustments practice of algorithmic stablecoin platforms. With unlimited liability, the platform is always able to perform these adjustments and, as a result, always maintains the peg. However, as we show next, the mere introduction of limited liability jeopardizes the platform's ability to always maintain the peg, even under full commitment.

3.2 Limited Liability

The full-commitment policy with unlimited liability requires that the platform conduct large stablecoin repurchases when the underlying cryptoasset value drops in order to restore an optimal demand ratio. For a large drop, however, the repurchase cost might exceed the post-repurchase platform value. In practice, the platform would then be unable to finance the entirety of repurchase necessary to maintain the peg by issuing new equity tokens, even if it is committed to full dilution of legacy equity token owners.

From this point, we assume that policies must satisfy limited liability. That is, the platform's equity value must be positive at all times.[22] In other words, no smart contract

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[22]The term "limited liability" typically refers to the legal protection provided to shareholders, whereby the company's liability does not extend beyond the company's assets. In this work, we use this term to refer to the fact that, in the anonymity of the blockchain, it is impossible to credibly commit to recapitalizing a

may impose actions such that the platform's continuation value is negative. The value of the platform under limited liability constraint at date t, $E_t \geq 0$, can be derived at each point in time through the same steps as in Proposition 2:

$$E_t = \mathbb{E}\left[\int_t^T e^{-r(s-t)}\left(\ell(A_s, C_s)C_s + (\mu^k - r)\varphi C_s\right)ds \,\middle|\, A_t, C_{t^-} = 0\right]\left(-(p_t - \varphi)C_{t^-} \geq 0. \quad (12)\right.$$

The first term in (12) is the total platform value from date t onward, as in (10) at date 0. The second term, $(p_t - \varphi)C_{t^-}$, captures the net value of outstanding debt. Note that this term is zero at date 0, as $C_{0^-} = 0$, so that it does not appear in (10). The equity value of the platform at time t is thus equal to the value of a new platform that starts with zero stablecoins, net of the cost of repurchasing all outstanding stablecoins. The effective repurchase cost per unit is given by $p_t - \varphi$, because buying back one stablecoin frees up collateral value φ. Equation (12) therefore suggests that collateral can help relax the limited liability constraint—an intuition we formalize below.[23]

First, we argue that for a large enough negative demand shock, the optimal policy with unlimited liability in Proposition 2 violates constraint (12). After a negative demand shock, the platform should repurchase a large stock of stablecoins to implement target $a^\star$. If the shock is large enough, however, this cost can exceed the present value of future convenience yields. In this case, the platform's equity value would then become negative if the platform were to implement the policy in Proposition 2. At that point, the platform is unable to finance the stablecoin buy-backs necessary to maintain the peg.

To analyze the platform's problem under full commitment and limited liability—i.e., problem (8) with additional constraint (12)—we focus on a set of policies defined below. To characterize these policies, it is useful to define the demand ratio $a_t \equiv A_t/C_{t^-}$.

Definition 2. *A target Markov policy (TMP) is given by demand ratio thresholds $\{\underline{a}, \overline{a}, a^\star\}$*

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platform beyond what is effectively pledged in a smart contract (including forced dilution of equity).

[23]The equity value decomposition in (12) does not imply that the platform must repurchase all stablecoins before issuing new ones. It simply breaks down any policy into two steps that occur simultaneously at the same price: (i) repurchase all outstanding stablecoins C_{t^-} and (ii) issue new stablecoins to the new level, C_t.

with $\underline{a} \leq \overline{a} \leq a^\star$; an interest rate policy $\delta_t = \delta(A_t, C_{t^-}) = \delta(a_t)$; and an issuance policy

$$d\mathcal{G}_t = \begin{cases} G(A_t, C_{t^-})dt & \text{if} \quad \underline{a} \leq a_t < \overline{a}, \\ \dfrac{A_t}{a^\star} - C_{t^-} & \text{if} \quad a_t \geq \overline{a}, \end{cases} \tag{13}$$

where the issuance policy over $[\underline{a}, \overline{a}]$ is said to be smooth (of order dt). The platform enters liquidation when the demand ratio is below $\underline{a}$.

We call the policies considered in Definition 2 Markov because they depend on the history of shocks and actions only via state variables A_t and C_{t^-}. This memoryless property considerably simplifies our analysis in the presence of constraint (12).[24] The optimal policy in the unlimited liability benchmark is a TMP with $\underline{a} = \overline{a} = 0$. Definition 2 generalizes this policy in two ways. First, a TMP may feature a *smooth region* $[\underline{a}, \overline{a}]$ in which the platform abandons the target and switches to an issuance policy of order dt—that is, it makes smooth adjustments to the stablecoin stock.[25] Second, there may also be a liquidation region below demand ratio threshold $\underline{a}$. As discussed above, relaxing the strict commitment to the target demand ratio may prove necessary to satisfy limited liability constraint (12). Target Markov policies may come with some loss of generality under full commitment and limited liability, however. Yet, when we relax commitment to stablecoin issuance in Section 4, we show that the optimal policy belongs to this class.

The platform's equity value and the stablecoin price inherit the Markov property of the platform's policies, denoted now by $E(A, C)$ and $p(A, C)$, and thereafter omitting the time index. Due to the homogeneity of the problem, the ultimate state variable for our problem is the demand ratio $a = A/C$, so we also define $e(a) \equiv E(A, C)/C$ and $p(a) \equiv p(A, C)$, where $e(a)$ is the platform's equity value per stablecoin outstanding. Using this notation, the platform's objective (8) can be rewritten as

$$E_0 = E(A, C^\star(A_0)) + (p(a^\star) - \varphi)C^\star(A_0) = A_0 \frac{e(a^\star) + p(a^\star) - \varphi}{a^\star}, \tag{14}$$

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[24]The general problem is not standard because limited-liability constraints (12) are forward-looking, which means equity value E_t is not the solution to a standard Hamilton-Jacobi-Bellman (HJB) equation. Techniques developed by Marcet and Marimon (2019) do not apply to our problem; the additional complexity comes from the term $(p_t - \varphi)C_{t^-}$ on the right-hand side of (12) as a state variable C_{t^-} multiplies forward-looking variable p_t, which depends on all future policy choices. Our focus on Markov policies ensures that the equity value and the stablecoin price solve HJB equations.

[25]See the definition provided by DeMarzo and He (2021, p. 1205).

with $C^\star(A) = A/a^\star$. The platform's objective is comprised of the sum of date-0 issuance gains, $(p(a^\star) - \varphi)C^\star(A_0)$, and the post-issuance equity value, $E(A, C^\star(A_0))$.

To solve for the optimal policy, equation (14) shows that we need to characterize the equilibrium equity value and the price at the target ratio $a^\star$. In our model, this ultimately requires solving for these functions over the whole state space. To do so, we guess and verify that the equilibrium price satisfies $p(a) = 1$ for $a \geq \bar{a}$ and $p(a) < 1$ for $a \in [\underline{a}, \bar{a})$, which implies that investors enjoy liquidity benefits only in the target region. We first show that the platform designs the policy in the smooth region $[\underline{a}, \bar{a}]$ so that the limited liability constraint binds when the peg is lost.

Lemma 1. *In the smooth region $[\underline{a}, \bar{a}]$, an optimal TMP under full commitment and limited liability satisfies $\delta(a) = 0$ and*

$$g(a) \equiv \frac{G(a, C_-)}{C_-} = -\frac{\mu^k \varphi}{p(a) - \varphi}, \tag{15}$$

that is, the platform does not pay interest when the peg is lost and uses all collateral proceeds to repurchase stablecoins. Under repurchase policy (15), $e(a) = 0$ for all $a \in [0, \bar{a}]$ and the price solves the following equation when $a \in [\underline{a}, \bar{a}]$

$$(r + \lambda)p(a) = (\mu - g(a))ap'(a) + \frac{\sigma^2}{2}a^2 p''(a) + \lambda \mathbb{E}[p(Sa)], \tag{16}$$

subject to the two boundary conditions $p(\underline{a}) = \varphi$ and $p(\bar{a}) = 1$.

The intuition for this result is as follows. As shown by (10), the platform has value to the extent it captures investors' liquidity benefits. As a result, the platform seeks to minimize the time it spends in the smooth region $[\underline{a}, \bar{a}]$, where the peg is lost ($p(a) < 1$) and investors enjoy no such benefit. To do so and increase the growth rate of $a_t = A_t/C_{t^-}$ when $a_t \in [\underline{a}, \bar{a}]$, stablecoin issuance is minimized in this region. This policy involves paying no interest to investors and using all the returns on collateral to buy back stablecoins. This condition yields equation (15), because each stablecoin is backed by collateral value φ that grows at rate μ^k. The net repurchase cost of a stablecoin is $p - \varphi$ because buying back a stablecoin frees up collateral value φ. Hence, equation (15) defines the maximum rate at which the platform can repurchase stablecoins while satisfying limited liability constraint $e(a) \geq 0$.

The second part of the Lemma confirms that the platform's equity value is zero when the

19

peg is lost. Intuitively, the platform would otherwise still have slack in the limited liability constraint to buy back stablecoins and defend the peg. Finally, Lemma 1 characterizes stablecoin price dynamics in the smooth region $[\underline{a}, \overline{a}]$ when the peg is lost. Its evolution is governed by HJB equation (16). Optimal repurchase policy (15) enters this equation because it governs the rate at which the demand ratio a_t increases in region $[0, \overline{a}]$.

Thanks to Lemma 1, we can characterize the equilibrium equity value under a TMP up to the level of the interest rate $\delta(a^\star)$ paid at the target ratio.

Lemma 2. *Under a TMP that satisfies Lemma 1, the platform's equity value under full commitment and limited liability is characterized by*

$$e(a) = \begin{cases} 0 & \text{if} \quad a \leq \overline{a}, \\ [e(a^\star) + (p(a^\star) - \varphi)] \frac{a}{a^\star} - (p(a^\star) - \varphi) & \text{if} \quad a \geq \overline{a}, \end{cases} \tag{17}$$

$$(r + \lambda - \mu)e(a^\star) = \mu^k \varphi + \mu(p(a^\star) - \varphi) - \delta(a^\star)p(a^\star) + \lambda \mathbb{E}[e(Sa^\star)], \tag{18}$$

where $\delta(a^\star)$ is the interest rate that maintains the peg at parity $p(a^\star) = 1$, defined by

$$\delta(a^\star) = r - \ell(a^\star) + \lambda \left(1 - \mathbb{E}[p(Sa^\star)]\right). \tag{19}$$

Characterization of the equity value when $a \leq \overline{a}$ follows directly from Lemma 1 and the fact that $e(a) = 0$ when the platform is in default $(a \leq \underline{a})$. Consider now the target region $[\overline{a}, \infty)$, in which the platform issues or repurchases stablecoins to maintain a constant demand ratio $a^\star$. By definition, the equity value is then given by

$$E(A, C) = E(A, C^\star(A)) + (p(a^\star) - \varphi)(C^\star(A) - C). \tag{20}$$

That is, the equity value in the target region is equal to the equity value at the target plus the net issuance proceeds (repurchase costs) when issuing (buying back) stablecoins to reach the target. Dividing both sides of (20) by the stablecoin stock C, we obtain (17).

Equation (18) characterizes the equity value at the target ratio $a^\star$. When at the target, equity holders receive interest on collateral $\mu^k \varphi$, issue new stablecoins at (expected) rate μ for a gain $p(a^\star) - \varphi$, and pay interest $\delta(a^\star)$. The term $-\delta(a^\star)p(a^\star)$ corresponds to the cost of buying back these stablecoins paid as interest in order to maintain the target

demand ratio $a^\star$. Finally, the last term on the right-hand-side of (18) corresponds to the expectation of a large (Poisson) negative shock to demand A.

Finally, equation (19) gives the interest rate paid by the platform at the target. For a given target ratio $a^\star$, the rate with limited liability exceeds that with unlimited liability, given by $r - \ell(a^\star)$, in the presence of Poisson shocks ($\lambda > 0$). Under limited liability, large negative demand shocks may force the platform to abandon the peg. To compensate for this expected devaluation, the platform must pay a higher interest rate relative to the case with unlimited liability. This feature suggests that platforms with high observed interest rates are less stable.[26]

Our last preliminary result is that default is not optimal under full commitment.

Lemma 3. *Under full commitment and limited liability, the platform never defaults:* $\underline{a} = 0$.

The platform never defaults because doing so cannot increase the platform's value at date 0. The result is most intuitive for an uncollateralized platform ($\varphi = 0$). If the platform defaults below some threshold $\underline{a} > 0$, the stablecoin price then falls permanently to 0. Since the demand ratio can fall below $\underline{a}$ following a large enough negative shock, default below $\underline{a}$ reduces investors' willingness to pay for the stablecoin at date 0. To maintain the peg, the platform would need to pay high interest $\delta(a^\star)$ at the target, as suggested by (19). This feature in turn depresses the platform's equity value, as can be seen from (18). If, instead, the platform keeps on operating when $a \leq \underline{a}$, it can wait for positive shocks to recover and regain the peg. This lowers the required interest rate $\delta(a^\star)$ relative to the case in which the platform defaults below $\underline{a}$, which increases equity value at the target $a^\star$.

The argument for Lemma 3 is more complex when the platform holds collateral ($\varphi > 0$), since default allows the platform to transfer collateral to users. Thanks to Lemma 1, however, we can show that the equilibrium price without default satisfies $p(a) \geq \varphi$ for all $a > 0$, which implies that such transfer does not increase the platform's ex ante value.[27] Overall, although the platform may lose the peg, it never defaults under full commitment.

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[26]Before it crashed, the Terra-Luna platform was offering interest rates above 20%.

[27]If $p(a) \leq \varphi$, it would be optimal for the platform to repurchase a stock of stablecoins at no additional cost than selling the corresponding collateral. Thus, there is a reflecting boundary at $\tilde{a}$ such that $p(\tilde{a}) = \varphi$ (i.e., $g(\tilde{a}) = -\infty$).

3.3   Optimal Platform Design

We now use our preliminary results to characterize the optimal policy as the solution to an optimization problem over the target ratio $a^\star$ and the lower bound $\bar{a}$. The last necessary step to rewrite objective function (14) as a function of these parameters only is to derive the interest rate at the target ratio $\delta(a^\star)$ as a function of $(\bar{a}, a^\star)$. This requires solving for the equilibrium price for $a \in (0, \bar{a})$. Unfortunately, we cannot provide a general analytical solution for the general case because of the feedback loop in equation (16) for the price via the issuance rate $g(a)$ given by (15).

Nonetheless, two special cases of interest allow for an explicit characterization of the platform's optimal policy: the uncollateralized case ($\varphi = 0$) and the fully collateralized case ($\varphi = 1$). We use these two extreme cases to study the effect of collateralization on platform stability and provide a numerical analysis for the general case. We first present results assuming an equilibrium exists and then state conditions for existence.

Purely Algorithmic Platforms   Consider first an uncollateralized platform with $\varphi = 0$. In this case, we obtain an analytic solution for the price thanks to equation (16), because Lemma 1 shows that $g(a) = 0$ for $a \in (0, \bar{a}]$ when $\varphi = 0$. We then obtain the following characterization of the optimal policy.

Proposition 3 (Purely Algorithmic Platform Equilibrium). *If an equilibrium with positive stablecoin value exists for an uncollateralized platform ($\varphi = 0$), then:*

1. *The region $[0, \bar{a}]$ in which the peg is lost is non-empty and the equilibrium stablecoin price is given by $p(a) = \left(\frac{a}{\bar{a}}\right)^{-\gamma}$, for $a \leq \bar{a}$ where $\gamma < -1$ is the unique negative root of*

$$r + \lambda = -\mu\gamma + \frac{\sigma^2}{2}(1+\gamma)\gamma + \frac{\lambda\xi}{\xi - \gamma}. \tag{21}$$

2. The optimal policy is characterized by Lemmas 1, 2, and 3 and $(\overline{a}, a^\star)$ that solve

$$\frac{e(a^\star) + p(a^\star)}{a^\star} = \max_{\overline{a}, a^\star} \left\{ \left(\frac{\ell(a^\star)/a^\star}{r + \frac{\lambda}{\xi+1} - \mu + \left(\frac{\lambda\xi}{\xi+1} - \frac{\lambda\xi}{\xi-\gamma} \right) \left(\frac{a^\star}{\overline{a}} \right)^{-(\xi+1)}} \right) \right. \tag{22}$$

$$subject \; to \qquad e(\overline{a}) = \left[e(a^\star) + p(a^\star) \right] \frac{\overline{a}}{a^\star} - 1 = 0. \tag{23}$$

An uncollateralized platform loses the peg after a large enough negative demand shock, even if it can commit to an issuance policy. When demand drops, the platform would like to repurchase a sufficient amount of stablecoins to maintain the peg. For a large enough drop, however, this repurchase cost exceeds the net present value of liquidity benefits. In that case, repurchases cannot be financed through equity dilution and the peg is lost. This result is analogous to Del Negro and Sims (2015) and Reis (2015) who show that an insolvent central bank without fiscal support is unable to control inflation.

In the region $[0, \overline{a}]$ in which the peg is lost, the stablecoin price remains strictly positive— although investors enjoy no liquidity benefit, since $p(a) < 1$. The stablecoin value is then driven entirely by the probability that the demand ratio a_t exogenously reaches the peg threshold $\overline{a}$ due to a series of positive demand shocks. The speed of this process depends on the value of the root γ.

The second part of Proposition 3 characterizes the optimal policy choice of an uncollateralized platform under limited liability. Given that $e(a) = 0$ for all $a \leq \overline{a}$ and $e(a)$ is linear and increasing for $a \in [\overline{a}, \infty)$, limited liability holds for all a if $e(\overline{a}) = 0$, which is constraint (23). As in the case with unlimited liability, the platform maximizes the present value of liquidity benefits. With limited liability, however, the platform's effective discount rate increases and depends on its policy choices, as shown by (22), because the platform may lose the peg. Given threshold $\overline{a} > 0$, a lower value of $a^\star$ increases the probability of losing the peg, which raises the platform's discount rate. Setting $\overline{a} = 0$ to kill this effect is not possible, because this would violate limited liability constraint (23)—that is, $e(\overline{a}) > 0$. This discount rate effect implies that the optimal target demand ratio $a^\star$ is higher than its counterpart with unlimited liability, $a^\star_{ul}$: Reducing stablecoin issuance from $C^\star_{ul}(A)$ to $C^\star(A) < C^\star_{ul}(A)$ protects the platform against large negative demand shocks.

We provide an illustration of our results in Figure 2 that contrasts the solutions under
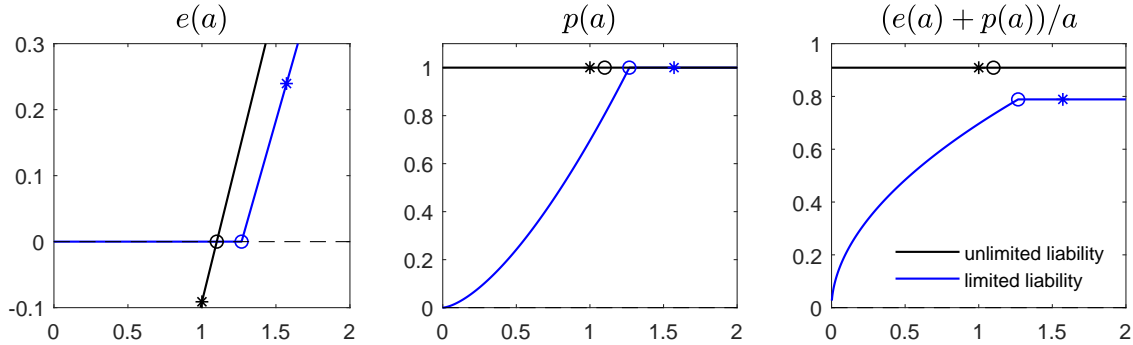
Figure 2: Full-commitment solution with limited liability without collateral (blue) and unlimited liability without collateral (black). The set of parameters is given by $r = 0.06$, $\mu = 0.05$, $\sigma = 0.1$, $\ell(A, C) = r \exp(-C/A)$, $\xi = 6$, $\lambda = 0.10$. Asterisks represent the target demand ratio $a^\star$ and circles indicate $\bar{a}$, the point at which $e(a)$ reaches zero.

limited and unlimited liability. The left panel shows that limited liability protects equity holders, since their equity value is always positive after large negative shocks. From an ex ante perspective, however, the inability to conduct large repurchases lowers the total platform value, as can be observed in the right-most panel. The center panel shows that the stablecoin price loses the peg at $\bar{a}$ under the limited liability case. These dynamics can be observed in the crash of the two algorithmic stablecoins, Terra and NuBits, for which the market capitalization of their governance tokens fell to zero at the time of losing the peg (see Appendix A).

Fully Collateralized Platforms   We now turn to the analysis of fully collateralized stablecoin platforms. The feature that simplifies the analysis in this case is that limited liability constraint (12) never binds because stablecoin repurchases are financed entirely from collateral holdings.

Proposition 4 (Fully Collateralized Platform Equilibrium). *If an equilibrium with positive stablecoin value exists for a fully collateralized platform, the following results apply:*

1. *The peg is always maintained; that is, $\bar{a} = 0$.*

2. *The optimal policy is given by Lemmas 1, 2, and 3, $\bar{a} = 0$ and $a^\star > a^\star_{ul}$ that solves*

$$\max_{a^\star} \frac{e(a^\star)}{a^\star} = \frac{\ell(a^\star) + \mu^k - r}{r - \mu + \lambda/(\xi + 1)} \frac{1}{a^\star}. \tag{24}$$

The key difference between a fully collateralized platform (Proposition 4) and an uncollateralized one (Proposition 3) is that the peg is never lost in the former; that is, $\bar{a} = 0$. This result follows directly from the observation that limited liability constraint (12) may never bind when $p(a^\star) = \varphi = 1$, because the net value of outstanding stablecoins is zero. With a fully collateralized platform any stablecoin repurchase is fully financed by collateral holdings, which means that no equity dilution is ever required to pay for a repurchase. Hence, the limited liability constraint does not affect the ability of the platform to perform these operations and maintain the peg under full collateralization.[28]

This observation implies that the optimization problem (24) is identical to that of a fully collateralized platform under unlimited liability. An important difference, however, is that with unlimited liability, the platform would choose not to hold collateral because it is costly ($\mu^k < r$) and large stablecoin repurchases would be financed by allowing equity to become negative. With limited liability, the platform must hold collateral to maintain the peg at all times. The comparison between target ratios $a^\star$ and $a^\star_{ul}$ reflects this additional collateral cost. Under limited liability, the platform issues fewer stablecoins because the net liquidity benefit per stablecoin that accounts for the collateral cost is lower than under unlimited liability, since $\ell + \mu^k - r \leq \ell$ given our assumption that $\mu^k \leq r$.

3.4 Existence Conditions

Propositions 3 and 4 characterize a platform's optimal policy, given that an equilibrium with positive stablecoin value exists. Now we provide existence conditions for both cases.

Proposition 5 (Existence Condition). *Given collateralization ratio $\varphi \in \{0,1\}$, a stablecoin platform with positive value exists under full commitment only if*

$$\max_a \ell(a) \geq \begin{cases} r - \mu + \frac{\lambda}{\xi+1} & if \quad \varphi = 0, \\ r - \mu^k & if \quad \varphi = 1. \end{cases} \tag{25}$$

We derive the existence conditions in Proposition 5 from imposing $e(a^\star) \geq 0$. The condition implied by $e(\bar{a}) \geq 0$ and $a^\star \geq \bar{a}$ is necessary and sufficient for an equilibrium

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[28]The result holds in part because the collateral asset is uncorrelated with the demand process A_t. If it were (positively) correlated, the value of collateral holdings would decrease when demand drops. In this case, the collateral available to finance a repurchase may fall short of the repurchase cost. Unless collateral is fully correlated with the demand process, however, the qualitative effect whereby collateral relaxes limited liability constraint (12) remains.
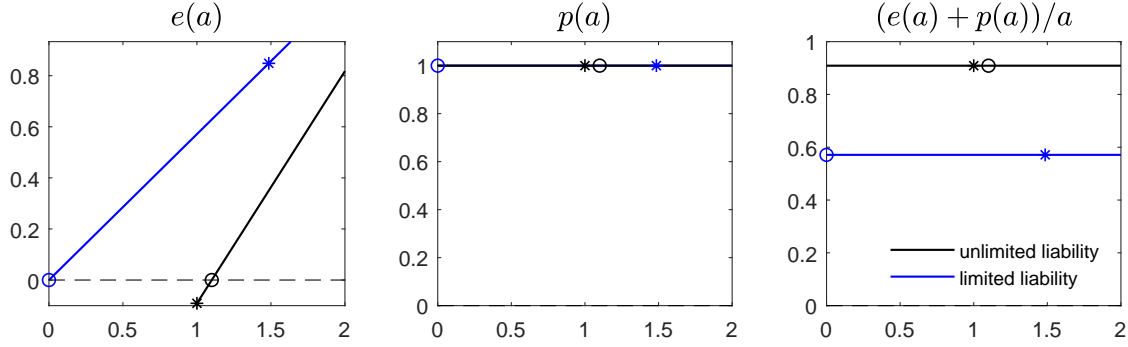
Figure 3: Full-commitment solution with limited liability fully collateralized (blue) and unlimited liability without collateral (black). The set of parameters is given by $r = 0.06$, $\mu = 0.05$, $\mu^k = 0.055$, $\sigma = 0.1$, $\ell(A, C) = r \exp(-C/A)$, $\xi = 6$, $\lambda = 0.10$. Asterisks represent the target demand ratio $a^\star$ while and circles indicate $\bar{a}$, the point at which $e(a)$ reaches zero.

with positive stablecoin value to exist under limited liability. We report this condition directly in the fully collateralized case ($\varphi = 1$), but only provide a necessary condition in the uncollateralized case ($\varphi = 0$) for simplicity. The proof contains a sufficient condition.

In the fully collateralized case ($\varphi = 1$), the existence condition states that the liquidity benefit captured by the platform $\ell(a^\star)$ must exceed the collateral holding cost $\mu^k - r$. As discussed above, the collateral holding cost can be interpreted as a liquidity benefit from holding the underlying asset, which the platform forgoes when using the asset as collateral. Condition (25) states that issuing stablecoins that are fully backed by another asset can only be profitable if the former commands larger liquidity benefits.

In the uncollateralized case ($\varphi = 0$), condition (25) follows from the requirement that the growth rate of stablecoin demand, $\mu - \lambda/(\xi + 1)$ must exceed the interest paid by the platform, which satisfies $\delta^\star \geq r - l(a^\star)$ by equation (19). Paying interest to users entails buying back stablecoins to maintain the demand ratio at the target.[29] Hence, the difference between the growth rate of demand and the interest rate is the net issuance rate of stablecoins. It must be positive for equity tokens to have any value. In other words, an uncollateralized platform can emerge only if stablecoin demand keeps growing over time.

Corollary 1. *An uncollateralized platform value can exist only if stablecoin demand grows; that is, if $\mu - \frac{\lambda}{\xi+1} \geq 0$.*

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[29]This component is absent for a fully collateralized platform because the net cost of buying back stablecoins is then 0.

Corollary 1 follows from Proposition 5 and Assumption 1 stating that the marginal liquidity benefit is no larger than the discount rate ($\ell < r$). Stablecoin demand must grow over time for an uncollateralized stablecoin platform to have any value. Without this growth component, platform owners' only dividend would be the cost of buying back stablecoins paid as interest to maintain the peg. As it emphasizes the importance of demand growth, this result relates to the existing argument on algorithmic stablecoins, which are portrayed as "Ponzi schemes". If the growth rate of the demand for the stablecoin were to unexpectedly and permanently fall to zero, the equity value of the platform would also fall to zero and the peg would be permanently lost.

3.5   Undercollateralized Platforms: Numerical Solution

Partially collateralized platform, $\varphi \in (0, 1)$, do not have analytical solutions. In Figure 4, we solve numerically for the optimal collateralization rule $\varphi^\star(\lambda)$ for different demand shock intensities λ. As λ goes up, the likelihood that limited liability constraint (12) binds increases together with the probability of a large negative shock. Collateral thus becomes more useful because a higher collateralization ratio φ relaxes constraint (12): The platform can finance purchases from collateral holdings to a greater extent when φ is high. In line with Proposition 5, the right panel of Figure 4 shows that collateral is necessary for a stablecoin platform to exist when negative shocks are likely enough (high λ). With full collateralization ($\varphi = 1$), a platform always exists for all values of λ, as shown above. In practice, there exists a large heterogeneity of platform designs, ranging from uncollateralized ones such as Terra-Luna to partially collateralized ones such as FRAX to fully collateralized ones such as DAI. Our model suggests that an optimal collateralization ratio trades off stability with platform profits.

4   Non-programmable Issuance

In this section, we analyze the centralized platform's problem under a weaker form of commitment. Unlike in Section 3, stablecoin issuance and repurchase cannot be fully programmed via smart contracts at date 0. We maintain, however, commitment with respect to the interest rate policy $\{\delta_t\}_{t \geq 0}$ and the minimum collateralization rule φ chosen
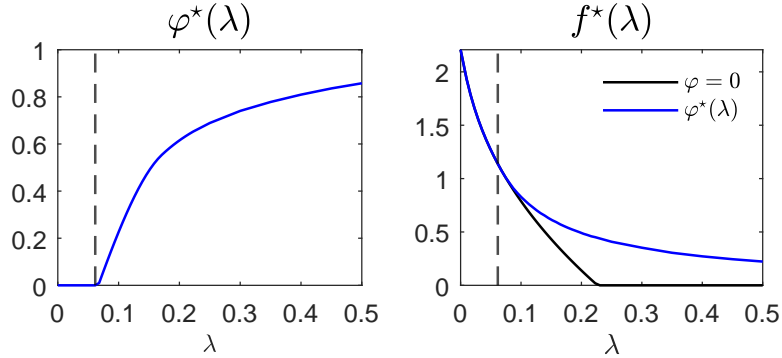
Figure 4: Full-commitment solution with limited liability. The function $\varphi^\star(\lambda)$ represents the optimal collateralization rate $\varphi^\star$ for different levels of large demand shock intensity λ. The function $f^\star(\lambda)$ represents the total platform value $(e(a^\star) + p(a^\star) - \varphi^\star)/a^\star$ at the optimal target demand ratio $a^\star$ and either optimal collateralization rate $\varphi = \varphi^\star$ (blue) or without collateral $\varphi = 0$ (black) for different levels of large demand shock intensity λ. The set of parameters is given by $r = 0.06$, $\mu = 0.05$, $\mu^k = 0.055$, $\sigma = 0.1$, $\ell(A, C) = r \exp(-C/A)$, $\xi = 6$. The numerical solution algorithm is described in the Internet Appendix.

at date 0.[30] Our analysis under limited commitment is motivated by the fact that many stablecoin protocols retain discretion over the repurchase and issuance of stablecoins in practice.[31]

In what follows, we first refine our equilibrium concept under partial commitment and highlight the new implementation constraints that arise in this case. Then, we characterize the optimal design of a centralized platform that cannot commit to its issuance policy.

4.1 Equilibrium Concept under Partial Commitment

We refine our equilibrium concept under partial commitment by considering Markov perfect equilibria (MPE), defined with respect to the state variables of our economy (A_t, C_{t^-}). In a MPE, the platform's issuance policy and the stablecoin pricing function depend only on (A_t, C_{t^-}), as opposed to the entire history of shocks. In line with our previous analysis, the coupon policy δ chosen at date 0 can only depend on the demand ratio $a_t = A_t/C_{t^-}$.

Definition 3. *Given a coupon policy $\delta(A, C)$ homogeneous of degree 0 and a collateraliza-*

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[30]Limited liability still applies. If equity token holders find the interest policy or collateralization requirements too costly ex post, they can liquidate the platform.

[31]As described below, some degree of commitment is necessary for an equilibrium with positive stablecoin value to exist. See Appendix C.1 for a proof of this claim.

tion ratio $\varphi \in [0, 1]$, a MPE is given by equity token value function $E(A, C)$; a stablecoin
pricing function $p(A, C)$; an issuance policy $d\mathcal{G}(A, C)$; and an optimal default policy τ such
that the issuance policy $d\mathcal{G}$ and default policy τ maximize the platform's equity value in any
state (A, C). That is,

$$E(A, C) = \max_{\tau, d\mathcal{G}} \mathbb{E}\left[\int_t^T e^{-r(s-t)} \left(p_s d\mathcal{G}_s + \mu^k \varphi C_s - \varphi dC_s\right) \, A_t = A, C_{t^-} = C\right], \qquad (26)$$

given the law of motion for stablecoins (4) and stablecoin pricing function

$$p(A, C) = \mathbb{E}\left[\int_t^T e^{-r(s-t)} (\ell_s + \delta_s) p_s ds + e^{-r(\tau-t)} \varphi \, A_t = A, C_{t^-} = C\right], \qquad (27)$$

where the expectation in (27) is taken over future platform policies.

Optimality criterion (26) states that the issuance policy must be sequentially optimal under limited commitment. In a Markov equilibrium, this means the policy must be optimal in any state (A, C). In writing equation (26) for the equity value above, we simplified equation (6) by substituting for the collateral repurchase policy $d\mathcal{M}_t = \varphi dC_t - \mu^k \varphi C_t$ implied by collateralization rule (3).

In a Markov equilibrium, the stablecoin price (27) may only depend on past actions of the platform via state variables (A, C). This means stablecoin users may not collectively punish the platform for deviating from an issuance policy they would agree upon at date 0. If instead investors could use "grim-trigger" strategies to punish the platform, additional outcomes could be supported. Our focus on Markov equilibria disciplines the analysis in that the stablecoin price may only depend on fundamentals and users' expectations about the platform's future policies. We note that enforcing collective punishments may prove challenging with disperse investors, as in our model.[32]

The first step of the analysis is to characterize an equilibrium stablecoin issuance policy $d\mathcal{G}$ and default policy τ under partial commitment. Remember that under full commitment, we posited that $d\mathcal{G}$ belongs to the class of targeted Markov policies (TMP). Under limited commitment, however, sequential optimality imposes stronger requirements on the equilibrium policy, which allow us to prove that it must belong to the TMP class.

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[32]See Malenko and Tsoy (2020), who consider punishments in a related dynamic leverage choice problem for firms. In their model, following a deviation from the equilibrium policy, the firm and investors play the MPE of DeMarzo and He (2021), which gives the lowest possible equilibrium payoff to the firm.

Proposition 6 (Equilibrium Policy). *For an optimal interest rate policy δ chosen at date 0, if an equilibrium exists, the equilibrium issuance policy $d\mathcal{G}$ under limited commitment belongs to the class of TMP introduced in Definition 2.*

The proof of Proposition 6 has several technical steps we briefly outline below. We first establish that the equilibrium equity function is weakly convex and the stablecoin price is weakly increasing as a function of the demand ratio a. Following arguments by DeMarzo and He (2021), we then show that the equilibrium issuance policy is smooth (features jumps) on intervals for which the equity value is strictly convex (linear). The existence of a default threshold $\underline{a}$ follows from the fact that the equity value is increasing in a. Next, we show that if the coupon policy $\delta(a)$ is chosen optimally at date 0, the issuance policy is smooth over the first part of the no-default region, $[\underline{a}, \overline{a}]$ for some $\overline{a} \geq \underline{a}$. For values of $a \in [\overline{a}, \infty)$, it features a jump to some target demand ratio $a^\star$. By definition, these results imply that the equilibrium policy belongs to the class of TMP.

4.2 Commitment Constraints

Similar to the full-commitment case, the platform chooses the policy parameters of the TMP at date 0. With limited commitment, however, the platform faces additional constraints because the issuance and default decisions must now be sequentially optimal. The following proposition characterizes these constraints generated by lack of commitment.

Proposition 7. *Under limited commitment, a feasible TMP and the equity value function of the induced nonzero MPE must satisfy the following properties:*

1. *The platform's issuance rate in the smooth region $[\underline{a}, \overline{a}]$ is given by*

$$g(a) = \frac{a\delta'(a)p(a) + (\mu^k - r)\varphi}{ap'(a)} \tag{28}$$

   and the platform's equity value is the same as if it issued no debt.

2. *The value of equity tokens must satisfy the following smooth-pasting conditions at the lower bound of the target region $\overline{a}$ and at the default threshold $\underline{a}$ if $\underline{a} > 0$, respectively:*

$$e'(\overline{a}) = \frac{e^\star + 1 - \varphi}{a^\star}, \tag{29}$$

$$e'(\underline{a}) = 0. \tag{30}$$

30

3. In the target region, for all $a \geq \bar{a}$, the interest rate must satisfy

$$\delta(a) - \delta(a^\star)\frac{a}{a^\star} \geq \left[(r+\lambda)(1-\varphi) + \mu^k\varphi\right]\left(1 - \frac{a}{a^\star}\right) + \lambda\left[\mathbb{E}[e(Sa)] - \mathbb{E}[e(Sa^\star)]\frac{a}{a^\star}\right] \tag{31}$$

Condition 1 of Proposition 7 characterizes the optimal issuance policy in the region $[\underline{a}, \bar{a}]$ with smooth stablecoin issuance. Smooth issuance is optimal only if the returns from issuance are equal to 0. We show in the proof that this condition is given by

$$p(a) - \varphi = e'(a)a - e(a). \tag{32}$$

Equation (32) states that the net marginal benefit of issuing stablecoin, $p(a) - \varphi$, is equal to the marginal loss of equity value from such issuance. The corollary whereby returns to issuance are zero in the smooth region is similar to the leverage ratchet effect of DeMarzo and He (2021) for a firm issuing debt. In their work, the leverage ratchet effect implies that the firm can never capture the tax advantage of debt, which is akin to the liquidity benefit in our model. While their result holds in the smooth region $[\underline{a}, \bar{a}]$, our equilibrium may also feature a target region $[\bar{a}, \infty)$, as we explain below. In the smooth region, the equity value is solved as if the platform issued no debt because issuance returns are zero . Equilibrium issuance is determined in equilibrium, however, to satisfy condition (32).

The equilibrium issuance policy in the smooth region has two components captured by the two terms at the numerator on the right-hand side of (28). First, the platform tends to repurchase stablecoin to reduce the amount of costly collateral it must hold, because $\mu^k < r$ by assumption. Second, the platform issues (repurchases) stablecoins if $\delta'(a) > 0$ ($\delta'(a) < 0$). In particular, if the interest rate decreases with a, $\delta'(a) < 0$, the interest rate policy induces the platform to repurchase stablecoins so as to increase its demand ratio.

Condition 2 gathers the standard smooth-pasting conditions the equity value must satisfy in equilibrium. Equation (29) ensures that the platform optimally switches from discrete repurchases in the target region to a smooth issuance policy at threshold $\underline{a}$. This condition requires that the derivative of the equity value is continuous at threshold $\bar{a}$. Equation (30) ensures that the liquidation threshold is optimally chosen by the platform.

Finally, Condition 3 ensures that implementing demand ratio $a^\star$ is ex post optimal without commitment when the platform is in the target region $a \in [\bar{a}, \infty)$. This condition

is thereby crucial to support an equilibrium with a target region. We obtain this condition by considering a "one-step" deviation whereby, starting from some demand ratio $a \neq a^\star$, the platform would remain idle during an interval of period dt and then revert back to the conjectured equilibrium policy.[33] To understand constraint (31), rewrite it as:

$$\delta(a)C - \delta(a^\star)C^\star(A) + \underbrace{\lambda(\mathbb{E}[E(SA, C^\star(A))] - \mathbb{E}[E(SA, C)])}_{\text{protection against large shocks}}$$

$$\geq \underbrace{(r + \lambda)(1 - \varphi)(C - C^\star(A))}_{\text{gains from postponing repurchase}} + \underbrace{\mu^k \varphi(C - C^\star(A))}_{\text{collateral dividends}}. \tag{33}$$

To fix ideas, suppose that the current stablecoin stock C is higher than the target $C^\star(A)$ so that the equilibrium policy is to repurchase $C - C^\star(A)$. The terms on the left-hand side of equation (33) represent the net advantages of adhering to the equilibrium policy relative to maintaining stablecoin stock C during a period dt before reverting back to the equilibrium policy. The first term, $\delta(a)C - \delta(a^\star)C^\star(A)$, represents the net interest savings from increasing the demand ratio from a to $a^\star$. The second term measures the relative protection against negative (Poisson) demand shocks at target ratio $a^\star$ relative to ratio $a < a^\star$. On the right-hand side, the two terms capture the relative benefits of a deviation, respectively, from delaying the repurchase of stablecoins and earning dividends on a larger stock of collateral, $\varphi C > \varphi C^\star(A)$, in the meantime.

Constraint (33) shows that promising a high interest rate in the target region for $a \neq a^\star$ can discipline the platform. Intuitively, the platform has more incentives to implement the target ratio $a^\star$ if it must deliver a large interest rate to users should it deviate. The notion that off-equilibrium punishments can sustain the implementation of the peg is intuitive but the punishment cost is endogenous here. Consider for instance an uncollateralized platform. While it faces no collateral cost from issuance, paying a high interest rate in stablecoins is costly because the platform's equity value is decreasing with the outstanding stock of stablecoins in equilibrium.[34][35] In fact, a state-contingent interest policy is necessary to

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[33]We show that this condition also rules out deviations such that the platform repurchases or issues debt smoothly during an interval dt.

[34]Our analysis of the deviation leading to condition (33) in the proof of Proposition 7 shows that the argument is similar for a collateralized platform. Although paying interest comes with a direct collateral cost, this cost is recouped by the platform when it reverts to the equilibrium policy.

[35]In DeMarzo and He (2021), the tax payment $\pi(Y - cF)$ *decrease* with more debt F. Thus, the requirement from condition (33) that the interest payment increases with more debt to deter from the gains from postponing repurchase cannot be satisfied.

sustain an equilibrium with a target ratio and a price peg.

Corollary 2. *With a noncontingent interest policy, $\delta(a) = \delta^\star$ for all a, there exists no nonzero MPE under limited commitment.*

This result is more striking for a fully collateralized platform which finances stablecoin repurchases entirely with collateral. To see why a state-contingent interest policy is still necessary, set $\varphi = 1$ and $\delta(a) = \delta(a^\star)$ for all a so that no-deviation condition (31) becomes

$$\forall a, \quad (\delta(a^\star) - \mu^k)\left(1 - \frac{a}{a^\star}\right) \gtreqless 0 \quad \Rightarrow \quad \delta(a^\star) = \mu^k, \tag{34}$$

where the term proportional to λ disappears because $E(A, C^\star(A)) = E(A, C)$ for all C with a fully collateralized platform (Lemma 2). Together with equation (19) to maintain the peg ($\delta(a^\star) = r - \ell(a^\star)$ when $\varphi = 1$), condition (34) implies that the platform's value is zero by Proposition 4 because the collateral cost $r - \mu^k$ then fully offsets the liquidity benefit $\ell(a^\star)$ captured by the platform. The platform finds the deviation profitable not because it wants to delay repurchase costs, which are zero with full collateralization, but because it can earn the interest spread $\mu^k - \delta(a^\star)$ on a larger stablecoin stock relative to the target level $C^\star(A)$.

4.3 Optimal Protocol under Limited Commitment

We now characterize the policy design problem of the platform under limited commitment. The objective of the platform is again to maximize its date-0 value, which is given by

$$\frac{e(a^\star) + 1 - \varphi}{a^\star} = \frac{\ell(a^\star) + (\mu^k - r)\varphi + \lambda\mathbb{E}[e(Sa^\star) + p(Sa^\star) - \varphi]}{r + \lambda - \mu}\frac{1}{a^\star}, \tag{35}$$

with e and p the equilibrium equity value and pricing functions. To obtain (35), we used equations (18) and (19). The platform thus chooses the TMP that maximizes (35) subject to limited liability, $e(a) \geq 0$, and to the implementation constraints derived in Proposition 7. To express (35) and the constraints as a function of policy parameters, we need to solve for the equilibrium value functions and the stablecoin price over the entire state space. In the target region $a \in [\bar{a}, \infty)$, we have $p(a) = 1$ by construction and $e(a)$ given by equation (17), as in the full-commitment case. In the smooth region, we show in the proof that the

dynamic equations for the equity value and the price are, respectively,

$$(r + \lambda)e(a) = (\mu^k - \delta(a))\varphi + \underline{\delta}(e(a) - ae'(a)) + \mu ae'(a) + \frac{\sigma^2}{2}a^2 e''(a) + \lambda\mathbb{E}[e(Sa)], \quad (36)$$

$$(r + \lambda)p(a) = \delta(a)p(a) + (r - \mu^k)\varphi + (\mu - \delta(a))ap'(a) + \frac{\sigma^2}{2}a^2 p''(a) + \lambda\mathbb{E}[p(Sa)]. \quad (37)$$

Analytical solutions for (36) and (37) cannot be obtained in the general case, which motivates us to impose some structure on the feasible set of TMPs.

Assumption 4. *The interest rate policy in the smooth region $[\underline{a}, \overline{a}]$ is given by $\delta(a) = \underline{\delta}$.*

Assumption 4 allows us to provide an analytical solution for the MPE equity value and price in the smooth region. Our analysis below suggests that this assumption is innocuous, because we find that the platform always chooses to set $\underline{\delta}$ as high as possible. This indicates that state contingency in δ in the smooth region would not increase the platform's value.

Thanks to Assumption 4, we thus guess and verify that the equity value and the stablecoin price have the following functional forms:

$$e(a) = \begin{cases} 0 & \text{if } 0 \le a < \underline{a}, \\ \underline{e} + \sum_{k=1}^{3} c_k a^{-\gamma_k} & \text{if } \underline{a} \le a < \overline{a}, \\ (e^\star + 1 - \varphi)a/a^\star - (1 - \varphi) & \text{if } a \ge \overline{a}, \end{cases} \quad (38)$$

$$p(a) = \begin{cases} \varphi & \text{if } 0 \le a < \underline{a}, \\ \underline{p} + \sum_{k=1}^{3} b_k a^{-\gamma_k} & \text{if } \underline{a} \le a < \overline{a}, \\ 1 & \text{if } a \ge \overline{a}, \end{cases} \quad (39)$$

where $\{\underline{p}, \underline{e}\}$ and $\{b_k, c_k\}_{k=1,2,3}$ are parameters and $\{\gamma_k\}_{k=1,2,3}$s are the roots of equation

$$r + \lambda - \underline{\delta} = -(\mu - \underline{\delta})\gamma + \frac{\sigma^2}{2}(1 + \gamma)\gamma + \frac{\lambda\xi}{\xi - \gamma}. \quad (40)$$

As in the full-commitment case, and even under Assumption 4, an analytic characterization of the solution is difficult. We thus focus again on two extreme cases of interest: a purely algorithmic protocol ($\varphi = 0$) and a fully collateralized protocol ($\varphi = 1$), and we provide numerical results for partially-collateralized protocols.

Purely Algorithmic Protocol

We first consider a purely algorithmic platform without collateral; that is, with $\varphi = 0$.

34

In this case, the optimization problem of the platform can be characterized as follows.

Proposition 8. *Under limited commitment, a non-collateralized platform never defaults, that is, $\underline{a} = 0$. It chooses $\underline{\delta}, \bar{a}, a^\star$ to maximize*

$$\frac{e(a^\star) + 1}{a^\star} = \frac{\ell(a^\star)/a^\star}{r + \frac{\lambda}{\xi+1} - \mu + \left(\frac{\lambda\xi}{\xi+1} - \frac{\lambda\xi}{\xi-\gamma}\right)\left(\frac{a^\star}{\bar{a}}\right)^{-(\xi+1)}} \tag{41}$$

$$subject\ to \qquad e(\bar{a}) \equiv \left[e(a^\star) + 1\right]\frac{\bar{a}}{a^\star} - 1 = -\frac{1}{1+\gamma} > 0, \tag{42}$$

where $\gamma < -1$ is the lowest negative root of (40)

The first result from Proposition 8 is that an uncollateralized platform never goes into liquidation. Under limited commitment, the platform controls its issuance and repurchase and it does not have to buy collateral to back interest payments if $\varphi = 0$. Hence, the option value from default is zero; that is, $\underline{a} = 0$. As a result, the sole difference relative to the full-commitment case is smooth-pasting constraint (42), which replaces non-negativity constraint (23). Comparing (23) and (42) shows that the latter is more stringent, because $\gamma < -1$. Thus, limited commitment reduces the platform's value.

We now characterize the optimal policy that solves the optimization problem presented in Proposition 8. The reduced-form variable γ depends only on the interest rate $\underline{\delta}$ and it is decreasing. Hence, choosing $\underline{\delta}$ is similar to choosing directly γ. This latter variable plays two opposite roles. First, as shown in the proof of Proposition 8, the total platform value in the region where the peg is lost increases with γ, because this variable governs the speed at which the platform exits the smooth region. On the other hand, decreasing γ allows the platform to extend the target region $[\bar{a}, \infty)$ over which the price is pegged, as can be seen from constraint (42). Overall, the second effect dominates because the platform's paramount objective is to maintain the peg to be able to capture liquidity benefits.

Proposition 9. *An optimal uncollateralized platform's policy under limited commitment features $\underline{\delta} = \infty$, and thus $\gamma = -\infty$ such that $e(a) = p(a) = 0$ for $a \in [0, \bar{a}]$. The optimal target ratio $a^\star$ is strictly higher than under full commitment and the existence conditions are tighter.*

The first result that the interest rate $\underline{\delta}$ in the smooth region should be high in contrasts to its counterpart under full commitment in Lemma 1. Under full commitment, the platform
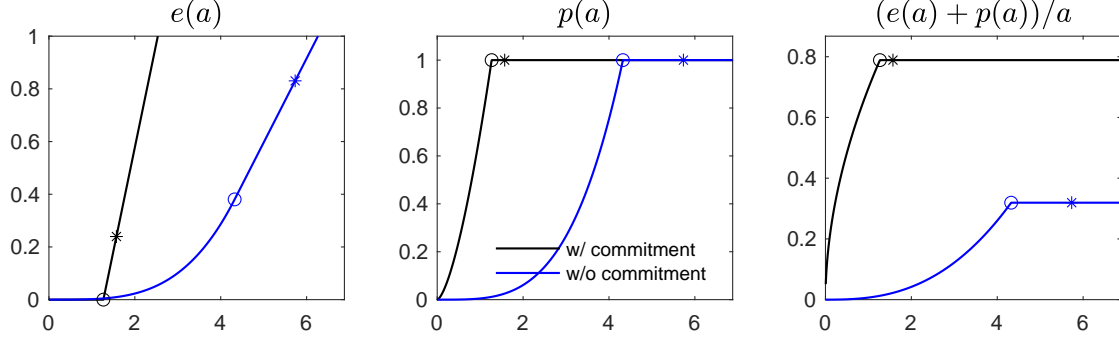
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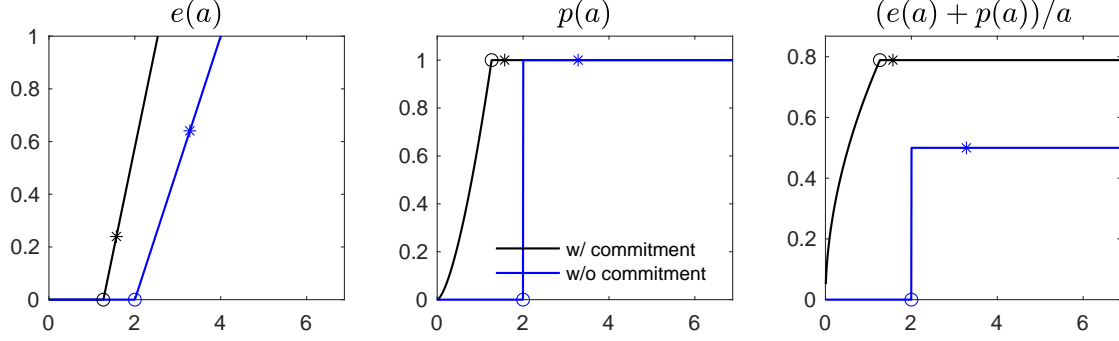
Figure 5: Purely algorithmic solution with commitment and limited liability (black) and without commitment (blue). The set of parameters is given by $r = 0.06$, $\mu = 0.05$, $\sigma = 0.1$, $\ell(A, C) = r \exp(-C/A)$, $\xi = 6$, $\lambda = 0.10$. Asterisks represent the target demand ratio $a^\star$ and circles indicate $\bar{a}$, the point at which $e(a)$ reaches zero.

minimizes the rate in the smooth region, which helps recover the peg. Under limited commitment, however, increasing $\underline{\delta}$ helps sustain incentives and increases the size of the target region. As stated above, the second effect dominates, which explain why $\underline{\delta} = \infty$ is now optimal. With $\underline{\delta} = \infty$, the stock of stablecoins jumps to $C = \infty$ whenever the platform enters the smooth region, which implies $e(a) = p(a) = 0$ for $a \in [0, \bar{a}]$. Hence, the platform designs the TMP such that it effectively liquidates once the peg is lost.

Proposition 9 also states that the target ratio is higher under limited commitment than under full commitment. A large negative shock to demand has worse consequences under limited commitment because it (optimally) triggers liquidation of the platform. Instead, under full commitment, the platform can recover after a shock that forces the platform to abandon the peg. This effect implies that the platform must be more conservative under limited commitment than under full commitment and thus issues fewer stablecoins for a given level of demand to accommodate large negative shocks.

We illustrate the results from Proposition 9 in Figure 5 where we compare the equity value, the stablecoin price and the total platform value to the case with full commitment. For completeness, we also derive numerically the optimal policy when the maximum interest rate $\underline{\delta}$ in the smooth region is bounded above. This constraint can capture implementation constraints or limits to the platform's ability to commit to paying an extremely high interest rate when it loses the peg. The results of this analysis are illustrated in Figure 6.

Fully Collateralized Platform We now turn to the analysis of a fully collateralized

36

Figure 6: Fully collateralized solution with commitment and limited liability (black) and without commitment (blue) constrained such that $\delta(a) \leq \mu$. The set of parameters is given by $r = 0.06$, $\mu = 0.05$, $\sigma = 0.1$, $\ell(A, C) = r \exp(-C/A)$, $\xi = 6$, $\lambda = 0.10$. Asterisks represent the target demand ratio $a^\star$ and circles indicate $\bar{a}$, the point at which $e(a)$ reaches zero.

protocol. In this case, we show that the full-commitment outcome can be implemented thanks to a state-contingent interest policy.

Proposition 10. *Under limited commitment and with full collateralization, interest rate rule $\delta(a) = r - \ell(a)$ implements the full-commitment outcome.*

With full collateralization, the TMP has no smooth region. Hence, conditions 1 and 2 of Proposition 7 are smooth. The commitment outcome can thus be sustained without commitment to the issuance policy if there exists an interest rate rule that satisfies condition (3). Plugging the rule of Proposition 10 in (34), we obtain the following condition:

$$\forall \, a, \quad \frac{\ell(a^\star) + \mu^k - r}{a^\star} \geq \frac{\ell(a) + \mu^k - r}{a}, \tag{43}$$

which holds by definition of $a^\star$, the optimal demand ratio target chosen at date 0. The intuition for the result is that the interest rate rule $\delta(a) = r - \ell(a)$ causes the platform to internalize ex post the cost of deviating from the ex ante profit-maximizing demand ratio. Hence, lack of commitment to the issuance policy has no impact on platform value.

To conclude this section, we provide numerical illustrations for total platform value with and without commitment as a function of the collateralization ratio φ in Figure 6. In both examples, the platform value is hump-shaped in φ and the optimal ratio is interior for the given parameters because collateral is costly. Without commitment, collateral benefits are larger, which explains why the optimal ratio is higher than with commitment. In line

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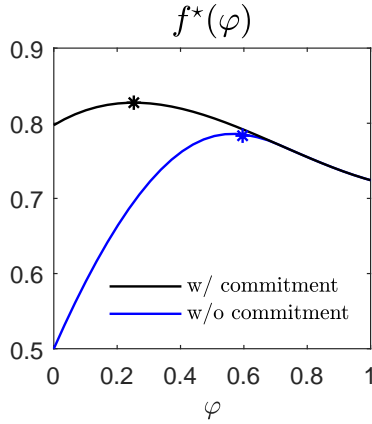
Figure 7: Undercollateralization solution with commitment and limited liability (black) and without commitment (blue). The function $f^\star(\varphi)$ represents the total platform value $(e(a^\star) + p(a^\star) - \varphi)/a^\star)$ at the optimal target demand ratio $a^\star$ for different collateralization rule φ. The set of parameters is given by $r = 0.06$, $\mu = 0.05$, $\sigma = 0.1$, $\ell(A,C) = r\exp(-C/A)$, $\xi = 6$, $\lambda = 0.10$. Asterisks represent the optimal collateralization rules. The numerical solution algorithm for the full-commitment solution is described in the Internet Appendix.

with Proposition 10, Figure 6 shows that the gap between the commitment and the no-commitment value narrows as the collateralization ratio increases because commitment can then be restored thanks to a state-contingent interest rate rule.

5   Decentralized Protocols

In this section, we extend our baseline framework to study *decentralized* stablecoin protocols. Such protocols—with DAI being the most prominent example—delegate the issuance of stablecoins to any users holding eligible collateral. Individual users who wish to issue stablecoins must lock some collateral asset in a smart contract generated by the protocol; this is termed a "vault." Once stablecoins are sold to outside investors, the vault represents a leveraged position in the collateral asset for its owner. Vault owners can unlock their collateral assets by repurchasing and "burning" enough stablecoins to liquidate the vault.

In decentralized protocols, all stablecoins are fungible, so the identity of the specific vault that issued the coin is irrelevant. The stability of a decentralized protocol therefore hinges

on the set of incentives provided by the platform to individual vault owners. In practice, equity token owners set the vaults' collateralization ratio and charge vault owners a fee intended to steer issuance and repurchase decisions by vault owners. As in the centralized case, equity token owners also set the interest rate paid to stablecoin users.

5.1 Decentralized Environment

We start by describing formally the features of a decentralized protocol. Any agent can open a vault and issue stablecoins, subject to the protocol rules. We index existing vaults by i and call C_t^i the amount of stablecoins outstanding for vault i at date t. The total stablecoin supply at date t is thus $C_t = \int_i C_t^i di$.

Equity token owners set a collateralization ratio $\varphi \in [0,1]$ for vault owners at date 0. At any date $t \geq 0$, token owners either shut down the platform or continue its operations, in which case they set a fee s_t charged to vault owners per unit of stablecoins issued and an interest rate δ_t paid to stablecoin users. The vault fee amounts to a tax on vaults: A vault owner with C_t^i stablecoins outstanding at date t must issue $s_t C_t^i$ extra coins, which it transfers to equity token owners. Of these $s_t C_t$ stablecoins it collects from vault owners, the platform transfers $\delta_t C_t$ units as interest payments to stablecoin users and sells the difference, $(s_t - \delta_t)C_t$, at market price p_t. As before, the stablecoin price is determined by competitive users:

$$p_t = \mathbb{E}\left[\int_t \left(e^{-r(s-t)} \left(\ell_s + \delta_s \right) p_s ds + e^{-r(\tau-t)} \varphi \right] \right. \tag{44}$$

with τ representing the platform liquidation date. Figure 8 illustrates flows in a decentralized protocol.

The decentralized protocol differs from our analysis of a centralized protocol in two key aspects. First, equity token owners delegate issuance to small vault owners and earn income from vault fees rather than from stablecoin issuance proceeds. Second, token owners choose the interest rate policy (and the vault fee) sequentially at every date t, rather than once and for all at date 0. This feature relaxes our earlier assumption that the platform commits to its interest rate policy. As we will show, these two features are connected: The delegated model with fee-based revenues provides commitment power to the platform.

The new building block in a decentralized protocol is the problem solved by individual
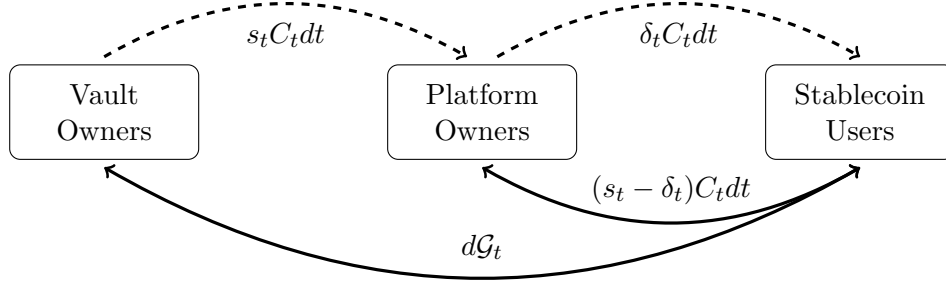
Figure 8: Stablecoin Flows in a Decentralized Platform. The dashed lines represent interest payments, and the solid lines represent market transactions at price p_t. s_t is the vault fee paid in stablecoins. δ_t is the interest payment paid in stablecoins to stablecoin users. $d\mathcal{G}_t$ is the aggregate issuance/redemption of stablecoins by vault owners.

vault owners. Taking as given the collateralization ratio φ, the price sequence $\{p_t\}_{t\geq 0}$, and the vault fee sequence $\{s_t\}_{t\geq 0}$, a vault owner i chooses its active supply $d\mathcal{G}_t^i$ and its default time τ^i. A vault owner with $C_{t^-}^i$ stablecoins outstanding at date t thus solves the following:

$$V_t^i(C_{t^-}^i) = \max_{\tau^i, d\mathcal{G}_t^i} \mathbb{E}_t\left[\int_t^{\tau^i \wedge \tau} e^{-r(s-t)}\left(p_s d\mathcal{G}_s^i - d\mathcal{M}_s^i\right)\right], \tag{45}$$

$$\text{subject to} \qquad dC_t^i = s_t C_t^i dt + d\mathcal{G}_s^i, \tag{46}$$

$$d\mathcal{M}_t^i = \varphi dC_t^i - \mu^k \varphi C_t^i dt. \tag{47}$$

A vault owner enjoys active issuance proceeds $p_s d\mathcal{G}_s^i$ net of collateral purchase $d\mathcal{M}_s^i$ until default or platform liquidation. In these events, vault owners receive nothing because the collateral value falls short of the stablecoins' par value as $\varphi \leq 1$. Equation (46) captures the law of motion for vault owners' outstanding stablecoins, and (47) is the law of motion for the vault's collateral value that ensures collateralization ratio $K_t^i = \varphi C_t^i$ is satisfied.

The optimization problem of equity token owners differs from (7) because they delegate stablecoin issuance. At every date t, they choose whether to default, and if not, the fee s_t charged to vault owners and the interest rate δ_t paid to stablecoin users. To do so, they solve the following:

$$E_t = \max_{\tau, \delta, s} \mathbb{E}_t\left[\int_t^{\tau} e^{-r(s-t)}\left((s_s - \delta_s)p_s C_s - \int_i C_s^i(1-\varphi)\mathbb{1}\{\tau^i = s\}di\right) ds\right], \tag{48}$$

40

subject to $C_t = \int_i C_t^i di$, where C_t^i is determined by optimization problem (45) for individual vault owner i, and given pricing equation (44). The equity token dividend is proportional to the difference between the fee s_t charged to vault owners and the interest δ_t paid to users. At every date t, equity token owners must also cover the collateral shortfall, equal to $C_t^i(1 - \varphi)$ for all liquidated vaults, if any, or shut down the system.

As in the centralized case, we analyze Markov equilibria of the decentralized model with state variables $a_t = \frac{A_t}{C_{t^-}}$. In addition to aggregate state variable a_t, each vault owner i considers its own stock of stablecoins $C_{t^-}^i$ as an idiosyncratic state variable when solving problem (45). In the interest of space, we do not provide a formal definition of a Markov equilibrium for the decentralized game because it is similar to that of Section 4. We use $V(a, C^i)$ to represent vault owner i's value function.

5.2 Vault Arbitrage

As a first step in our analysis, we derive arbitrage relationships imposed by competitive stablecoin issuance from vault owners. Then, in Section 5.3, we use these relationships to characterize the platform's problem in a decentralized environment.

We begin by showing that the value of a vault is equal to the value of the collateral locked in the vault minus the value of the stablecoins issued by that vault. To see this, observe that a vault owner with C^i stablecoins can adjust holdings to any $\tilde{C}^i$ and receive net issuance benefits $p(A, C) - \varphi$ per unit of stablecoin. By definition of the value function, we thus have, for every $(C^i, \tilde{C}^i)$,

$$V(a, C^i) \geq V(a, \widetilde{C}^i) + (p(a) - \varphi)(\widetilde{C}^i - C^i). \tag{49}$$

The same relationship must hold, inverting C^i and $\tilde{C}^i$, which implies that (49) must hold as an equality so that the value function is linear in the amount of stablecoins issued. The key element driving this result is that each atomistic vault owner takes the price as given when issuing stablecoins. Next, given free entry for vault owners, an empty vault must have zero value: $V(A, C, 0) = 0$. Combined with (49), we get that

$$V(A, C, C^i) = \varphi C^i - p(a)C^i. \tag{50}$$

Equation (50) thus establishes that the value of a vault is equal to the value of the collateral

41

held minus the value of stablecoins outstanding. Intuitively, any "franchise value" from owning a vault would be competed away by new vaults. Given that the vault value is linear in C^i, we derive $V(a, C^i) = v(a)C^i$, with $v(a)$ representing the vault value per stablecoin.

The characterization of a vault value in (50) leads to the following preliminary result.

Lemma 4. *In a decentralized environment with active vaults, an equilibrium stablecoin price must satisfy $p(a) \leq \varphi$. Vault owners prefer emptying their vaults over defaulting.*

The proof is immediate. Equation (50) shows that a vault has positive value only if $p(A, C) \leq \varphi$. This inequality reflects an arbitrage constraint: The equilibrium price of a stablecoin cannot exceed the value of the collateral backing it. If instead $p(A, C) > \varphi$, vault owners could achieve an unbounded profit from issuing stablecoins. The second part of Lemma 4 follows. Upon default, a vault owner obtains zero payoff because the par value of stablecoins weakly exceeds the collateralization ratio $\varphi \leq 1$. Vault owners thus prefer buying back stablecoins to unlock collateral because it generates a net payoff $\varphi - p(a) \geq 0$ per stablecoin outstanding.

Next, we relate the vault value to the fee charged by the platform and the stablecoin price dynamics. As we show formally in the proof of Proposition 11, the return on a vault is given by

$$rv(a) = r(\varphi - p(a)) = \varphi \mu^k - \mu^p(a)p(a) - s(a)p(a), \tag{51}$$

where $\mu^p(a) \equiv \mathbb{E}[dp(a)/(p(a)dt)]$ is the expected growth rate of the stablecoin price. The middle equality follows from (50). A vault owner holds a long position in the collateral and a short position in stablecoins. Hence, a vault's return is equal to the return on collateral, $\varphi \mu^k$, minus the stablecoin price appreciation, $\mu^p(a)p(a)$, and the fee charged by the platform, $s(a)p(a)$.

Equation (51) formalizes a second arbitrage condition that must hold in an equilibrium with active vaults. If the fee $s(a)$ is too high (too low), competitive vault owners would close their vaults (issue an infinite amount of stablecoins). To understand the former outcome, suppose the left-hand side of (51) is strictly above the right-hand side. Then, the opportunity cost of a vault during period dt exceeds the flow payoff from the vault. Vault owners would then close their vaults by buying back stablecoins and unlocking collateral.

Finally, using the same notation as above, we reproduce the usual competitive pricing

equation for stablecoins. For any demand ratio $a =$,

$$rp(a) = \ell(a)p(a) + \delta(a)p(a) + \mu^p(a)p(a). \tag{52}$$

The return on a stablecoin is the sum of the liquidity benefit, $\ell(a)p(a)$, the interest paid by the platform, $\delta(a)p(a)$, and the price appreciation, $\mu^p(a)p(a)$.

5.3   Platform Policy and Decentralized MPE

We turn to the platform's policy choices. At date 0, it sets the collateralization ratio, φ, for vaults and chooses sequentially the interest rate, δ, and the vault fee, s. In choosing these variables, the platform takes into account arbitrage conditions (50), (51), and (52), which can be viewed as implementation constraints.

First, a decentralized platform must be fully collateralized. Lemma 4 shows that an equilibrium stablecoin price must satisfy $p(a) \leq \varphi$. Hence, given that $\varphi \in [0,1]$, a decentralized platform can maintain the peg at one for some values a in equilibrium only if $\varphi = 1$.

The platform's rental income is constrained by arbitrage conditions (51) and (52). Combining these two equations and setting $\varphi = 1$, we find that the platform's utility flow is

$$(s(a) - \delta(a))p(a)C = \ell(a)p(a)C + (\mu^k - r)C \quad \forall a. \tag{53}$$

Equation (53) is intuitive: With competitive users and vault owners, the platform captures all gains from trade net of collateral costs. The platform ultimately maximizes its rental income, given by (53), which entails maintaining the peg, $p(a) = 1$, as otherwise users enjoy no liquidity benefits. The stablecoin stock that maximizes the right-hand side of (53) is the same amount that a fully collateralized centralized platform would choose. While the platform directly issues stablecoins in the centralized model, it controls issuance from vault owners via the vault fee in the decentralized model. The main result of this section characterizes the optimal vault fee and the MPE of the decentralized protocol, given below.

Proposition 11 (Decentralized Protocol Equilibrium). *The nonzero MPE with a decentralized protocol implements the full-commitment outcome under full collateralization*

($\varphi = 1$) with a stablecoin stock given by

$$C^\star(A) = \arg\max_C \left\{ \ell(A, C)C + (\mu^k - r)C \right. .$$ (54)

To implement $C^\star(A)$, the platform sets a vault fee of the form

$$s(a) - \delta(a) = \begin{cases} \ell(a) + (\mu^k - r)/p(a) + \varepsilon & \text{if} \quad C_t > C^\star(A_t), \\ \ell(a^\star) + \mu^k - r & \text{if} \quad C_t = C^\star(A_t), \\ \ell(a) + (\mu^k - r)/p(a) - \varepsilon & \text{if} \quad C_t < C^\star(A_t), \end{cases}$$ (55)

where ε is strictly positive. The equilibrium interest rate is $\delta(a^\star) = r - \ell(a^\star)$.

In a decentralized protocol, the platform steers issuance toward its optimal target under full collateralization $C^\star(A)$, with the vault fee $s(a)$. To implement the desired target, the platform sets a vault fee contingent on the stock of stablecoins. Suppose, for instance, that the stock of stablecoins is too low, that is, $C_t < C^\star(A_t)$. The platform then lowers the fee so that vault owners issue stablecoins until C_t reaches $C^\star(A_t)$. At this point, the vault fee is such that vault owners are indifferent about issuance. In equilibrium, these adjustments occur instantaneously so that $s(a)$ for $a \neq a^\star$ is an off-equilibrium fee schedule.

The main result from Proposition 11 is that a decentralized protocol can implement the full-commitment outcome under full collateralization. This result is obtained without commitment to the interest rate policy for a decentralized protocol. In contrast, such commitment is crucial in centralized protocols as demonstrated in Proposition 9. Due to vault owner arbitrage, the type of deviation that would be optimal with centralized issuance under a non-state-contingent interest rule, $\delta(a) = \delta(a^\star)$, is no longer implementable.

To see this, consider again a deviation to $C_t = C_{t^-} < C^\star(A_t)$ after a negative shock to A_t. We showed in Corollary 2 that equity token holders of a centralized platform would benefit from this deviation because their net flow payoff is equal to the return on collateral minus the interest paid on stablecoins, $\mu^k - \delta(a^\star)$. As this flow is positive, equity token holders earn more income with a larger stock of stablecoins outstanding and thus resist reducing this stock to the target level $C^\star(A_t)$. This deviation cannot be implemented in a decentralized protocol. Stablecoin pricing equation (44) shows that if $\delta(a) = \delta(a^\star)$ and $a_t < a^\star$, we must have $\mu^p(a) > 0$ and thus $p(a) < 1$. This implies that the right-hand side of arbitrage equation (51) is strictly negative if $s(a) = \mu^k$, which means C_{t^-} cannot be an equilibrium outcome. Specifically, the threat that vault owners would shut down

vaults unless the platform decreases s below μ^k disciplines equity token holders. Overall, vault owners' arbitrage implies that equity token holders' net payoff is always equal to the total (flow) gains from trade, as shown by (53), which guarantees that the ex ante profit-maximizing decisions are also ex post optimal.

This result can be interpreted as an implementation of Coase (1972)'s leasing solution for a durable-good monopolist. Similarly, a centralized platform that sells stablecoins suffers from a time-consistency problem. Without commitment, it does not internalize the effect of the current issuance of stablecoins on the liquidity benefits of users who previously bought stablecoins. As Calvo (1978) already noted, a monetary authority that can earn seignoriage revenues tends to "overprint" money. The intuition from Coase (1972), which was formalized by Bulow (1982), is that the time-consistency problem is solved if the monopolist switches from selling to renting goods because all goods are then repriced every period. A decentralized protocol implements this leasing solution as it transforms the platform's gains into a rental income flow equal to the vault fee minus the interest paid to users.[36]

6 Conclusion

This paper analyzes the optimal design of stablecoin protocols. We examine the merits and vulnerabilities of various tools used to peg the stablecoin price in a model in which we take the stablecoin issuers' incentive problem seriously. Our analysis shows that partially collateralized platforms are always vulnerable to large demand shocks, even under full commitment. The optimal collateralization level thus trades off resilience against these shocks with collateral holding costs. Our model also suggests that partially collateralized platforms that pay high interest rates are more fragile. Finally, we analyze stablecoin protocols in which stablecoins are issued by independent vault owners, a crucial feature of many decentralized stablecoin designs such as DAI. We show that a decentralized design in which the platform charges a rental fee to vault owners solves time-consistency problems, although only if decentralized issuance is fully collateralized. To focus on our main research

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[36]A centralized platform directly renting stablecoins to users would be closer to the solution envisioned by Coase (1972). In the context of our paper, such a rental market can work only if anonymous stablecoin users have incentives to return stablecoins to the platform every period. The platform could ensure compliance by requiring users to post collateral. Our model of a decentralized protocol is equivalent to this scheme except that it separates vault owners who post collateral and issue stablecoins from investors who use stablecoins.

question, we assumed a reduced-form liquidity benefit for stablecoins and considered the problem of a single stablecoin issuer. We leave this microfoundation and the analysis of stablecoin competition to future research.

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Appendices

A   Stablecoins in the Midst of the 2022 Crypto Crash

This appendix provides a short introduction to the variety of stablecoin pegging mechanisms in practice, with an emphasis on their performance during the crypto crunch of May 2022. We review two custodial (USD Coin and Tether), a purely algorithmic (Terra), an overcollateralized (DAI), and an undercollateralized (FRAX) stablecoin platform. At the beginning of May 2022, these five stablecoins accounted for more than 80% of the total stablecoin market.

USD Coin

USD Coin (USDC) is a custodial (fully collateralized) stablecoin managed by the Centre consortium on behalf of the peer-to-peer payment technology Circle headquartered in Boston, MA. USDC effectively acts as a narrow bank by backing its stablecoins exclusively with cash (bank deposits or equivalents) and short-term Treasury securities and providing full redemption. During the May 2022 crypto crash, USDC fared particularly well, as can be seen in Figure 9: It maintained its peg, and the quantity of USDC outstanding increased during that time period. Given its conservative reserves management strategy, USDC presumably benefited from a "flight to safety" because investors were fleeing from fast depreciating crypto-currencies and other stablecoins.

Tether

Tether (USDT) is another custodial stablecoin that is a native of the Ethereum ledger and issued by Tether Limited company, which is domiciled in Hong Kong under the umbrella of Tether Holdings Limited in the British Virgin Islands. Although Tether claims to be "fully backed by US dollar reserves," its definition of reserves appear to be less restrictive than the one applied by USDC, and also includes privately issued commercial paper and corporate bonds but also volatile crypto-currencies.[37] Griffin and Shams (2020) report

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[37]Since 2021 and a $41 million fine by the Commodity Futures Trading Commission for misleading claims that it was fully backed by the US dollar, Tether Holdings Limited regularly reports a reserves audit from Cayman-based auditing companies.

suspicious transaction patterns on the blockchain and suggest that the platform has been using unbacked Tether creation to purchase large quantities of Bitcoin to support its price.

Figure 9 displays the time-series price of Tether and quantities outstanding. We can observe a sharp reduction in supply around the crypto crash of May 2022, along with a temporary depegging. Tether nonetheless reanchored within a couple of days and has so far proven to be able to absorb the $5 bn of redemption it has faced.

Terra

Terra (UST) is a prime example of a fully algorithmic (uncollateralized) stablecoin. As described in the main text, algorithmic stablecoins such as Terra are uncollateralized and rely exclusively on quantity adjustments through smart contracts that specify rules for stablecoin issuances and buybacks. In the case of Terra, these are ruled through an external module that allows any investor to exchange 1 unit of stablecoin (Terra) for 1 dollar's worth of governance token (Luna) and vice versa. Between its introduction in early 2020 and the crypto crash of May 2022, Terra was one of the fastest-growing stablecoin platforms. By May 2022, the quantity of stablecoin Terra outstanding was close to $20 bn while the governance token Luna had a peak market capitalization of $40 bn.

As can be seen in Figure 10, the platform completely collapsed between May 7 and May 12, 2022. In the right panel of Figure 10, we see how the platform attempted but failed to defend the peg. On May 12, the platform burnt around 8 bn of Terra, partly through the issuance of additional Luna at an exponential pace. As can be seen in the left panel, this massive issuance of Luna led to the complete freefall of its price to zero. Simultaneously, the Terra Foundation liquidated around $3 bn of Bitcoin it had held in reserves. Given the size of the shock, these adjustments were not sufficient to reanchor the peg, and the value of Terra eventually also fell very close to zero.

DAI

DAI is a fully decentralized, overcollateralized stablecoin platform. Because of its decentralized nature, DAI is slightly more complex than other stablecoins and requires a longer description. With DAI, every user is able to deposit some Ethereum-based crypto-asset as collateral in a smart contract called a collateralized debt position (CDP). The user can then issue and sell DAI stablecoin tokens against this collateral up to a certain

overcollateralization threshold while effectively retaining an equity tranche in the CDP. In doing so, CDP users acquire a leveraged position in the collateral asset. Initially, it was only possible to use Ethereum as a collateral asset, but the platform migrated to a multiple collateral system at the end of 2019. Since then, the custodial stablecoin USD Coin (see above) has been used extensively as collateral for DAI. To close the CDP and retrieve the locked collateral, the owner has to repurchase and burn all previously issued DAI from the secondary market.

The platform also issues its own governance token, Maker (MKR). Holding Maker allows the user to vote on key policies of the platform and effectively confers the right to future seigniorage revenues. The platform is able to generate revenues for Maker holders by collecting "stability" fees from CDP owners. These fees accrue to a "buffer" fund up to a certain limit and are then distributed to Maker holders as dividends.

The pegging mechanism in DAI is tied to its overcollateralization. When the collateral in a CDP falls below the required threshold, the position is automatically liquidated and collateral assets are sold in an auction to burn corresponding DAI. When auction proceeds are insufficient to repurchase all DAI issued by the CDP, new Makers are automatically issued to cover the shortfall. As shown in Figure 10, we can see that this mechanism was at play during the May 2022 crypto market crash. The platform then liquidated for \$3 bn worth of collateral in CDPs in order to burn more than \$2 bn worth of DAI. This process was nonetheless done in an orderly fashion, and parity was maintained throughout. As can be seen from the right-most panel, no additional Maker was required to be issued.

FRAX

Frax (FRX) is an undercollateralized platform that can be thought of as a hybrid between Terra and DAI. As with Terra, users can exchange the stablecoin FRX for the platform's governance token Frax Shares (FRS) and the converse. Because the platform is partly collateralized, the swap module requires that users bring both FRS and collateral in a given proportion. For instance, if the collateralization ratio is 90% and Frax is trading for more than 1 USD, users can exchange 90 USD Coins and \$10 worth of FRS in exchange for 100 Frax and sell them for a profit. The collateralization ratio in Frax is automatically reduced in expansion and increased in contraction, so that with a large surge in issuance, Frax would converge to a fully algorithmic platform like Terra.

In early May 2022, Frax had a collateralization rate of 86.75%. As can be seen in Figure 10, the platform managed to burn around a $1 bn without breaking its peg.
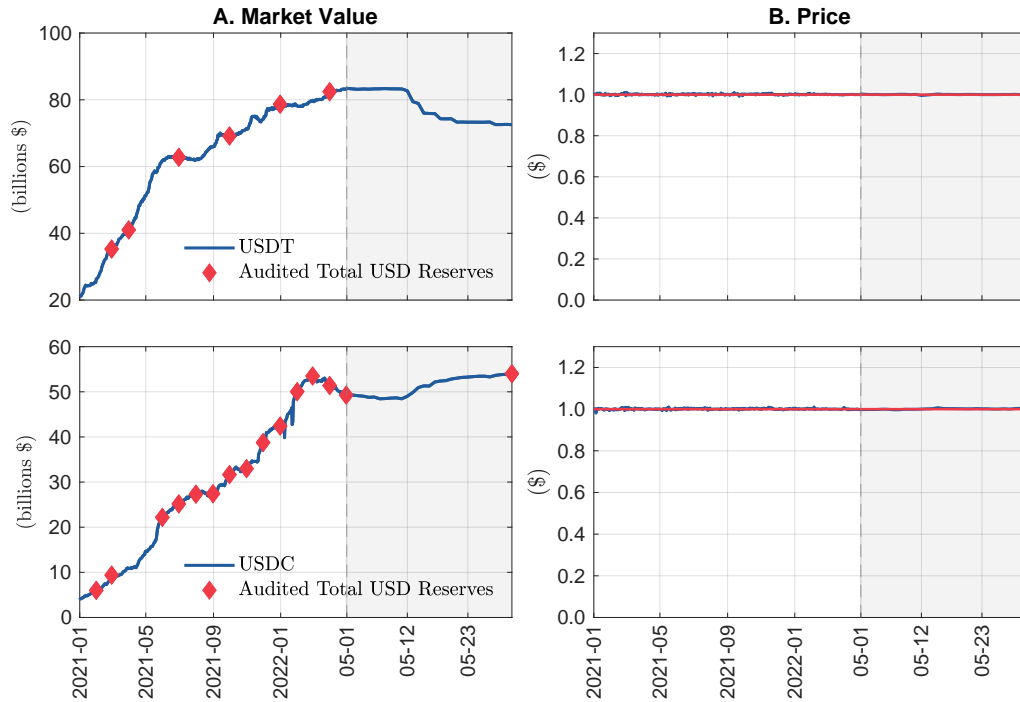
Figure 9: Custodial Stablecoins Time Series. This figure illustrates the daily time series of market capitalization and price for Tether (USDT, first row) and USD Coin (USDC, second row). The first portion of each graph spans the period from January 2021 to April 30 2022, while the gray shaded area zooms in on May 2022. Pink diamond markers in Panels A illustrate the total USD value of reserves backing the stablecoin, as certified through external audits made available on the platforms' respective web pages. Data sources: Market capitalization and prices are all retrieved through the CoinGecko API.
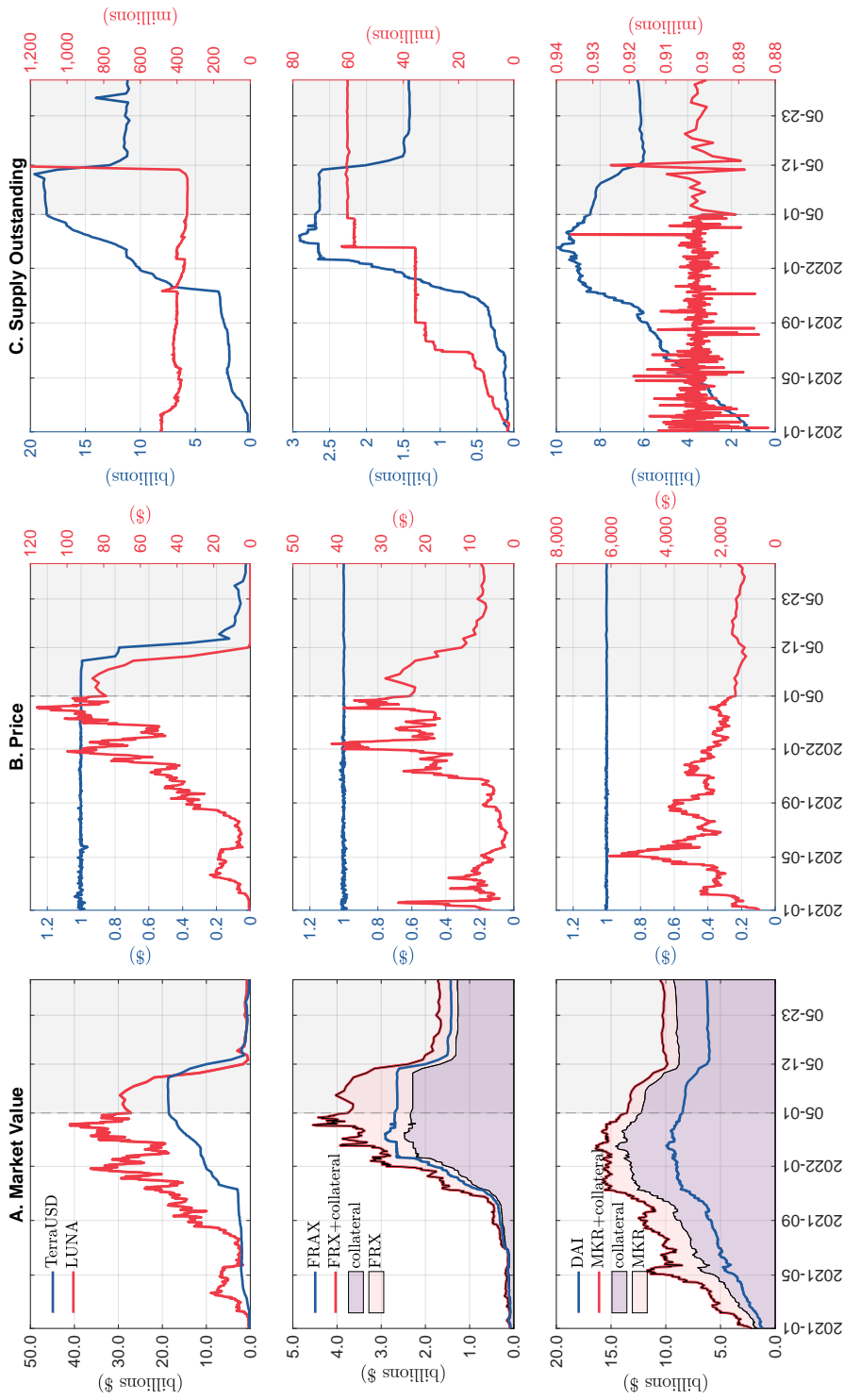
Figure 10: Algorithmic Stablecoins Time Series This figure illustrates the daily time series of market capitalization, price, and circulating supply, as denoted in each column title, for three algorithmic stablecoins. The first portion of each graph spans the period from January 2021 to April 30 2022, while the gray shaded area zooms in on May 2022. Each row plots the dynamics for a given stablecoin, as labeled in the first column; the blue solid line refers the stablecoin asset, while the pink solid line (or light pink shaded area in Panels A) refer to the corresponding governance token. Data sources: Market capitalization, prices, and supply outstanding are all retrieved through the CoinGecko API. The total USD value of FRAX collateral, illustrated in light shaded violet in the second row, was manually collected from https://app.frax.finance. The amount of DAI collateral was obtained by aggregating across all collateral assets, using the time-series debt data made available on Dune Analytics by @adcv via https://dune.com/queries/865375; we apply an adjustment factor to account for underestimating measurement error and impute the historical USD value of DAI collateral, illustrated in light violet in the last row, by rescaling the series by the ratio of Total DAI Locked ($) from https://daistats.com#/overview to the aggregated collateral series, both observed as of July 11, 2022, assuming a constant scaling factor.

55

B   Proofs

B.1   Proof of Proposition 2

Substituting for $d\mathcal{G}_t = dC_t - \delta_t C_t dt$, the objective function can be written as

$$E_0 = \max_{\varphi, \{\delta_t, d\mathcal{G}_t\}_{t \geq 0}} \mathbb{E}_0 \left[\int_0^\infty e^{-rt} \left(p_t dC_t - \delta_t p_t C_t dt + \mu^k \varphi C_t dt - \varphi dC_t \right) \right] \tag{B.56}$$

Integrating the terms in dC_t by parts, we obtain

$$E_0 = \max_{\varphi, \{\delta_t, d\mathcal{G}_t\}_{t \geq 0}} \mathbb{E}_0 \left[\left[(p_t - \varphi) C_t e^{-rt} \right]_0^\infty - \int_0^\infty e^{-rt} C_t \left(dp_t - r(p_t - \varphi)dt + \delta_t p_t dt - \mu^k \varphi \right) \right] \tag{B.57}$$

$$= \max_{\varphi, \{\delta_t, d\mathcal{G}_t\}_{t \geq 0}} \mathbb{E}_0 \left[\int_0^\infty e^{-rt} \left(\ell(A_t, C_t) \mathbb{1}_{\{p_t = 1\}} + (\mu^k - r)\varphi \right) C_t dt \right]. \tag{B.58}$$

To obtain the second line, we guess and verify that $\lim_{t \to \infty} \mathbb{E}_0[(p_t - \varphi)C_t e^{-rt}] = 0$. We use the pricing equation (6) to substitute for $dp_t - (r - \delta)p_t dt$ within the expectation.

Equation (B.58) shows that setting $\varphi = 0$ is optimal. Second, δ_t is only determined to the extent that it maintains the price peg, and we can rewrite equation (B.58) as

$$E_0 = \max_{\{\delta_t, d\mathcal{G}_t\}_{t \geq 0}} \mathbb{E}_0 \left[\int_0^\infty e^{-rt} \ell(A_t, C_t) C_t dt \right] \tag{B.59}$$

Assuming that such interest rate policy can be chosen, the platform's problem is static and the optimal issuance rule is such that C_t maximizes $\ell(A_t, C_t)C_t$. By Property (iii) in Assumption 1, this maximizer exists, is unique, and is given by (9). The fact that $C_{ul}^\star(A) = A/a_{ul}^\star$ is linear in A follows from Assumption 1. Moreover, our conjecture $\lim_{t \to \infty} \mathbb{E}_0[(p_t - \varphi)C_t e^{-rt}] = 0$ and the fact that the objective function is bounded follows from the fact that A_t grows at a rate inferior to r. Finally, the interest rate policy must be such that $p_t = 1$ for all t, which holds with $\delta(a_{ul}^\star) = r - \ell(a_{ul}^\star)$.

To conclude, the optimal issuance-repurchase policy $\{d\mathcal{G}_t\}_{t \geq 0}$ features a jump from 0 to $C_{ul}^*(A_0)$ at date 0 and is such that $d\mathcal{G}_t + \delta_t C_t dt = dA_t$ for $t > 0$.

B.2 Proof of Lemma 1

We guess and verify throughout that $p(a) = 1$ if and only if $a \in [\bar{a}, a^\star]$ and $p(a) < 1$ otherwise. This implies that liquidity benefits are enjoyed by stablecoin users only when $a \in [0, \bar{a}]$. We anticipate the result in Lemma 3 and conjecture an equilibrium with no platform default. We thus set $\underline{a} = 0$ and later prove in Lemma 3 that this feature is optimal. We proceed in three steps. We first show that $e(a) = 0$ is optimal for all $a \leq \bar{a}$ (Step 1). We then derive the optimal issuance policy in the smooth region (Step 2). Finally, we derive the HJB equation for the price in that region (Step 3).

Step 1. Total Platform Value

Consider first the net platform value F. Suppose $a = A/C > \bar{a}$. In this, case F only depends on A—not on the outstanding stock of stablecoins C—and we denote $\bar{F}(A)$ to avoid confusion. Let τ_S denote the first (stochastic) time when a shock $S \leq \underline{a}/a^\star$ hits. We have

$$\bar{F}(A_0) = E_{\tau_S}\left[\int_0^{\tau_S} e^{-rt}\left(\ell(A_t, C^\star(A_t))C^\star(A_t) + \varphi(\mu^k - r)C^\star(A_t)\right)dt \right.$$
$$\left. + e^{-r\tau_S}\mathbb{E}\left[F\left(SA_{\tau_S}, C^*(A_{\tau_S})\right)\ Sa^* \leq \bar{a}\right]\right]. \tag{B.60}$$

Given values for $(a^\star, \bar{a})$, maximizing value $\bar{F}(A_0)$ consists in maximizing the second term of the above equation. We thus make explicit the dynamic equation for $F(A, C)$ in the region where $a = A/C \in [0, \bar{a}]$. For a given $a \in [0, \bar{a}]$, denote $\tau(a)$ the first stochastic time when $a_t = \bar{a}$. We have

$$F(A_0, C_0) = \mathbb{E}_{\tau(a_0)}\left[\int_0^{\tau(a_0)} e^{-rt}(\mu^k - r)\varphi C_t dt + e^{-r\tau(a_0)}\bar{F}(A_{\tau(a_0)})\right] \tag{B.61}$$

subject to (1), $dC_t = (\delta_t C_t + G_t)dt.$ \tag{B.62}

The dividend flow for the total platform is negative in the region $[0, \bar{a}]$. Hence, maximizing $F(A, C)$ in region $[0, \bar{a}]$ and thus $\bar{F}(A)$ amounts to minimizing the expected time $\tau(a)$ from any given point a. Given the policies in $[0, \bar{a}]$ in (13), we have

$$\mathbb{E}\left[\frac{da_t}{a_t}\right] = \left(\mu - \frac{\lambda}{\xi + 1}\right)dt - (\delta_t + G_t/C_t)dt. \tag{B.63}$$

Hence the platform should seek to minimize δ_t and G_t subject to the constraint whereby equity $E(A, C)$ remains positive for $A/C \in [0, \bar{a}]$. Below, we determine lower bounds on δ_t and G_t compatible with this constraint.

Step 2. HJB for Equity Value

In the next step, we derive the recursive equation for the equity value in order to pin down the minimum value of $G(A, C)$ such that limited liability holds in region $[0, \bar{a}]$. In doing so, we guess and verify that it holds for $[\bar{a}, \infty)$. Adapting Equation (7), we have

$$
\begin{aligned}
E(A, C) =& (p(A, C) - \varphi)G(A, C)dt \\
&+ (1 - rdt)(1 - \lambda dt)\mathbb{E}[E(A + dA, C + dC) + \mu^k K dt + \varphi G(A, C)dt - \varphi dC] \\
&+ (1 - rdt)\lambda dt \mathbb{E}[E(SA, C)],
\end{aligned} \tag{B.64}
$$

where the terms within the first expectation operator correspond to the difference between the passive and active increases in collateral value ($\mu^k K dt + \varphi G(A, C)dt$) and the change in collateral value required to back the issuance of stablecoins (φdC). Using Ito's Lemma for the term $E(A + dA, C + dC)$ above and keeping only terms of order dt, we obtain the following HJB:

$$
\begin{aligned}
(r + \lambda)E(A, C) =& (p(A, C) - \varphi)G(A, C) + \mu A E_A(A, C) + \frac{\sigma^2}{2} E_{AA}(A, C) \\
&+ (\delta(A, C)C + G(A, C))E_C(A, C) + (\mu^k - \delta(A, C))\varphi C + \lambda \mathbb{E}[E(SA, C)].
\end{aligned} \tag{B.65}
$$

We rewrite the equation above as a functional equation for $e(a) = E(A, C)/C$. With $E_A(A, C) = e'(a)$, $E_{AA}(A, C) = e''(a)$, $E_C(A, C) = e(a) - ae'(a)$, and $g(a) \equiv G(A, C)/C$, we get

$$
\begin{aligned}
(r + \lambda)e(a) =& (p(a) - \varphi)g(a) + \mu a e'(a) + \frac{\sigma^2}{2}e''(a) + (\delta(a) + g(a))(e(a) - ae'(a)) \\
&+ (\mu^k - \delta(a))\varphi + \lambda E[e(Sa)].
\end{aligned} \tag{B.66}
$$

It follows from the equation above that the minimum value of $g(a)$ such that $e(a) \geq 0$ for all $a \in [0, \bar{a}]$ is given by

$$
g(a) = -\frac{\mu^k - \delta(a)}{p(a) - \varphi}\varphi.
$$

Given policy $g(a)$ above and $e(a) = e'(a) = 0$, the impact of $\delta(a)$ is offset in the HJB and we can set $\delta(a)$ to its minimum at 0 for $a \leq \bar{a}$. This concludes the proof.

Step 3. HJB equation for stablecoin price

Next, we characterize the price dynamics in region $[0, \bar{a}]$. The price equation can be written as

$$p(A, C) = (1 - rdt)(1 - \lambda dt)\mathbb{E}[p(A + dA, C + dC)] + (1 - rdt)\lambda dt \mathbb{E}[p(SA, C)]. \quad \text{(B.67)}$$

When $a \in [0, \bar{a}]$, stablecoin owners enjoy no cash flow because the platform optimally sets $\delta(a) = 0$ and liquidity benefits are equal to 0 because the price is not pegged to 1, since $p(a) < 1$. Using $dC = g(a)Cdt$, the first term on the right-hand side can be expanded using Ito's Lemma:

$$\mathbb{E}[p(A + dA, C + dC)] = p(A, C) + p_A(A, C)\mu A dt + \frac{\sigma^2}{2}A^2 p_{AA}(A, C)dt + p_C(A, C)g(a)Cdt$$

$$= p(a) + (\mu - g(a))ap'(a)dt + \frac{\sigma^2}{2}a^2 p''(a)dt. \quad \text{(B.68)}$$

To obtain the second line, we use the homogeneity of degree 0 of the price function, that is, $p(A/C) \equiv p(A, C)$, to replace $p_A(A, C) = p'(a)/C$, $p_{AA}(A, C) = p''(a)/C^2$ and $p_C(A, C) = -p'(a)A/C^2$. Plugging in (B.68) into (B.67) and keeping only terms of order dt, we obtain

$$0 = -(r + \lambda)p(a) + (\mu - g)ap'(a) + \frac{\sigma^2}{2}a^2 p''(a) + \lambda \mathbb{E}[p(Sa)] \quad \text{(B.69)}$$

which is equivalent to equation (16).

Finally, the boundary condition $p(\bar{a}) = 1$ obtains by construction in our conjectured equilibrium with $p(a) = 1$ for $a \geq \bar{a}$. Issuance policy (15) implies that $p = \varphi$ is a reflecting boundary. Hence, we have $p(a) > \varphi$ for all $a > \underline{a}$ and $p(\underline{a}) = \varphi$. This concludes the proof.

B.3 Proof of Lemma 2

The fact that equity value is equal to 0 in region $[\underline{a}, \bar{a}]$ is shown in Lemma 1. Because the collateralization ratio satisfies $\varphi \leq 1$, the platform's value is also equal to 0 in default region $[0, \underline{a}]$.

Consider now interval $[\bar{a}, \infty)$. As argued in the main text, by definition of a policy in

(13), equation (20) must hold. We can rewrite this relationship as follows:

$$Ce(a) = C^\star(A)e(a^\star) + (p(a^\star) - \varphi)(C^\star(A) - C). \tag{B.70}$$

Dividing both terms by C and using $C^\star(A) = A/a^\star$ by definition of $a^\star$, we obtain equation (18).

We are thus left to derive the HJB for the equity value at demand ratio $e(a^\star)$. The recursive equation is the following:

$$\begin{aligned} E(a^\star C_-, C_-) =& (1 - rdt)(1 - \lambda dt)\mathbb{E}\left[E(a^\star C_- + dA, C_- + dC) + \mu^k Kdt - \varphi dC \mid dN_t = 0 \right] \\ & + (1 - rdt)\lambda dt\mathbb{E}\left[E(Sa^\star C_-, C_-) \mid dN_t = 1 \right], \end{aligned} \tag{B.71}$$

where the term on the first line corresponds to the case in which the adjustment in demand A_t is smooth ($dN_t = 0$), while the second term corresponds to the case in which demand is hit by a Poisson shock ($dN_t = 1$). The term $\mu^k Kdt - \varphi dC$ corresponds to the change in collateral value.

We develop the first term corresponding to Brownian shocks. In region $[\bar{a}, \infty)$, we have

$$E(A, C) = C^\star(A)e(a^\star) + (p(a^\star) - \varphi)(C^\star(A) - C) = \left[e(a^\star) + p - \varphi\right]\frac{A}{a^\star} - (p - \varphi)C. \tag{B.72}$$

Hence, given that $dC = \delta(a^\star)Cdt$, we obtain the following relationship by Ito's Lemma:

$$\begin{aligned} \mathbb{E}\left[E(a^\star C_- + dA, C_- + dC) \mid dN_t = 0 \right] =& E(a^\star C_-, C_-) + \mu\left[e(a^\star) + p - \varphi\right]C^\star(A)dt \\ & - (p - \varphi)\delta(a^\star)C^\star(A)dt. \end{aligned} \tag{B.73}$$

Keeping only terms of order at least dt and dividing by $C^\star(A)$, we obtain

$$e(a^\star) = e(a^\star) + \left(-(r + \lambda)e(a^\star) + \mu\left[e(a^\star) + p(a^\star) - \varphi\right] - p(a^\star)\delta(a^\star) + \mu^k\varphi + \lambda\mathbb{E}[e(Sa^\star)]\right) dt,$$

which simplifies to equation (18). For future reference, we further solve for $e(a^\star)$ by

computing the term $\mathbb{E}[e(Sa^\star)]$. Using (17), we get

$$\mathbb{E}[e(Sa^\star)] = \int_0^{m(a^\star/\bar{a})} e(e^{-s}a^\star)\xi e^{-\xi s}ds \tag{B.74}$$

$$= \int_0^{m(a^\star/\bar{a})} \left[(e(a^\star) + p(a^\star) - \varphi)e^{-s} - p(a^\star) + \varphi\right]\xi e^{-\xi s}ds \tag{B.75}$$

$$= \frac{\xi}{\xi+1}\left(1 - \left(\frac{a^\star}{\bar{a}}\right)^{-(\xi+1)}\right)(e(a^\star) + p(a^\star) - \varphi) - \left(1 - \left(\frac{a^\star}{\bar{a}}\right)^{-\xi}\right)(p(a^\star) - \varphi). \tag{B.76}$$

Plugging this equation in (18), we get

$$\left(r + \frac{\lambda}{\xi+1} - \mu + \frac{\lambda\xi}{\xi+1}\left(\frac{a^\star}{\bar{a}}\right)^{-(\xi+1)}\right)e(a^\star)$$

$$= \left(\mu^k - \mu + \frac{\lambda}{\xi+1} - \lambda\left(\frac{a^\star}{\bar{a}}\right)^{-\xi} + \frac{\lambda\xi}{\xi+1}\left(\frac{a^\star}{\bar{a}}\right)^{-(\xi+1)}\right)\varphi$$

$$+ \left(\mu - \delta(a^\star) - \frac{\lambda}{\xi+1} - \frac{\lambda\xi}{\xi+1}\left(\frac{a^\star}{\bar{a}}\right)^{-(\xi+1)} + \lambda\left(\frac{a^\star}{\bar{a}}\right)^{-\xi}\right)p(a^\star). \tag{B.77}$$

Using $p(a^\star) = 1$, after some manipulations, we can rewrite the objective function as follows:

$$\frac{e(a^\star) + 1 - \varphi}{a^\star} = \frac{(\mu^k - r)\varphi + r - \delta(a^\star) + \lambda(1-\varphi)\left(\frac{a^\star}{\bar{a}}\right)^{-\xi}}{r - \mu + \frac{\lambda}{\xi+1} + \frac{\lambda\xi}{\xi+1}\left(\frac{a^\star}{\bar{a}}\right)^{-(\xi+1)}}. \tag{B.78}$$

Finally, we derive the value of the interest paid by the platform at the target ratio $a^\star$ in order to maintain the peg. To do so, we derive the dynamic equation for the price at the target. We have

$$p(A, C^\star(A)) = (\delta(a^\star) + \ell(A, C^\star(A)))p(A, C^\star(A))dt$$

$$+ (1 - rdt)(1 - \lambda dt)\mathbb{E}[p(A + dA, C^\star(A) + dC)] + (1 - rdt)\lambda dt\mathbb{E}[p(SA, C)]. \tag{B.79}$$

Note that from any point in target region $[\bar{a}, a^\star]$, the platform jumps discretely to $a^\star$. Hence, the equilibrium price is $p(a) = p(a^\star)$ when $a \in [\bar{a}, a^\star]$. We can thus replace $\mathbb{E}[p(A +$

61

$dA, C^\star(A) + dC)]$ with $p(a^\star)$. Keeping only terms of order dt, we obtain

$$(r + \lambda)p(a^\star) = (\delta(a^\star) + \ell(a^\star))\, p(a^\star) + \lambda\mathbb{E}[p(Sa^\star)].$$

Setting $p(a^\star) = 1$ and solving for $\delta(a^\star)$ we get (19).

B.4 Proof of Lemma 3

Equation (B.78) shows that for given $\{\bar{a}, a^\star\}$, the objective function depends on $\underline{a}$ only via the term $-\delta(a^\star)$, which is itself increasing with $\mathbb{E}[p(Sa^\star)]$. Next, $-\delta(a^\star)$ also enters positively the limited liability constraint $e(a^\star) \geq 0$, which implies that increasing $\mathbb{E}[p(Sa^\star)]$ also allows us to relax the constraint. Overall, the platform should set the default threshold $\underline{a}$ to maximize $\mathbb{E}[p(Sa^\star)]$. Below, we show that $\underline{a} = 0$ is the optimum.

Suppose the platform does not default; that is, $\underline{a} = 0$. Because the price $p(a)$ must be increasing for $a \in [0, \bar{a}]$ and $p(0) = \varphi$, it follows that $p(a) \geq \varphi$ for all $a \in [0, \bar{a}]$. Defaulting at some threshold $\hat{\underline{a}} > 0$ implies that the price would satisfy $p(a) = \varphi$ for all $a \in [0, \hat{\underline{a}}]$, which is weakly less than the price when $\underline{a} = 0$. Hence, default cannot increase the price on the interval $[0, \bar{a}]$ and thus cannot increase $\mathbb{E}[p(Sa^\star)]$. Indeed, the price of a stablecoin in $[\underline{a}, \bar{a}]$ is given by $p_t = \mathbb{E}_t\left[e^{-r(\tau-t)}(\mathbb{1}\{a_\tau \geq \bar{a}\} + \varphi\mathbb{1}\{a_\tau \leq \underline{a}\})\right]$ where $\tau \equiv \inf\{s \geq t; a_s \leq \underline{a} \cup a_s \geq \bar{a}\}$. This expectation is strictly decreasing in the default threshold $\underline{a}$. This proves that setting default threshold $\underline{a} = 0$ is optimal.

B.5 Proof of Proposition 3

Step 1. Our conjecture for the pricing function is

$$p(a) = \begin{cases} \sum_{k=1}^{3} b_k a^{-\gamma_k} & \text{if } 0 \leq a < \bar{a}, \\ 1 & \text{if } a \geq \bar{a}. \end{cases} \tag{B.80}$$

The issuance policy in region $[0, \bar{a}]$ is given by (15); that is, $g = 0$ when $\varphi = 0$. We first derive conditions on $\{\gamma_k\}_{k=1,2,3}$ such that HJB equation (16) is satisfied by our guess. We

have

$$p'(a) = -\sum_{k=1}^{3} b_k \gamma_k a^{-(\gamma_k+1)}, \tag{B.81}$$

$$p''(a) = \sum_{k=1}^{3} b_k \gamma_k (\gamma_k + 1) a^{-(\gamma_k+2)}, \tag{B.82}$$

$$\mathbb{E}[p(Sa)] = \int_0^\infty p(e^{-s}a)\xi e^{-\xi s}ds = \int_0^\infty \sum_{k=1}^{3} b_k e^{s\gamma_k} a^{-\gamma_k}\xi e^{-\xi s}ds = \sum_{k=1}^{3} \frac{b_k \xi a^{-\gamma_k}}{\xi - \gamma_k}. \tag{B.83}$$

Replacing into (16) and equalizing terms proportional to $a^{-\gamma_k}$, we obtain that for each $k \in \{1,2,3\}$, γ_k must be a root of characteristic equation (21). The roots of this third-order polynomial are

$$\gamma_k = -\frac{1}{2t_1}\left(t_2 + \zeta^\nu R + \frac{\Delta_0}{\zeta^\nu R}\right) \tag{B.84}$$

where

$$\Delta_0 = t_2^2 - 3t_1 t_3, \quad \Delta_1 = 2t_2^3 - 9t_1 t_2 t_3 + 27t_1^2 t_4,$$

$$R = \sqrt[3]{\frac{\Delta_1 + \sqrt{\Delta_1^2 - 4\Delta_0^3}}{2}}, \quad \zeta = \frac{-1 + \sqrt{-3}}{2}, \quad \nu = \{0,1,2\},$$

$$t_1 = -\frac{\sigma^2}{2}, \quad t_2 = \mu + \frac{\sigma^2}{2}(\xi - 1), \quad t_3 = -\mu\xi + \frac{\sigma^2}{2}\xi + r + \lambda, \quad t_4 = -r\xi.$$

According to Descartes' rule of sign, this polynomial has 2 positive roots and 1 negative root. Furthermore, using Budan-Fourier theorem, we can show that the negative root is strictly lower than -1. As shown in Corollary 1, $\mu - \lambda/(\xi+1) \geq 0$ is a necessary condition for a non-zero MPE to exists. Because the price must be bounded below by 0, the coefficients b_k, which correspond to positive roots must be 0. We now call γ the negative root of this polynomial.

The price function is thus given by $p(a) = ba^{-\gamma}$ for a $\in [0, \bar{a}]$. To determine b, we use the continuity of p at $\bar{a}$. Setting $p(\bar{a}) = 1$ yields $b = \bar{a}^\gamma$.

Step 2. We now show that the maximization problem of the platform at date 0 is given

by (22). Rewriting equation (B.77), we obtain

$$e(a^\star) + p(a^\star) = \frac{r - \delta(a^\star) + \lambda \left(\frac{a^\star}{\bar{a}}\right)^{-\xi}}{r + \frac{\lambda}{\xi+1} - \mu + \frac{\lambda\xi}{\xi+1}\left(\frac{a^\star}{\bar{a}}\right)^{-(\xi+1)}} p(a^\star). \tag{B.85}$$

We are left to substitute for $\delta(a^\star)$ thanks to equation (19). We have

$$\delta(a^\star) = r - \ell(a^\star) + \lambda\left(1 - \mathbb{E}[p(a^\star S)]\right),$$

$$= r - \ell(a^\star) + \lambda - \lambda \left[\int_0^{\ln(a^\star/\bar{a})} \xi e^{-\xi s} ds + \int_{\ln(a^\star/\bar{a})}^\infty \left(\frac{a^\star}{\bar{a}}\right)^{-\gamma} e^{s\gamma} \xi e^{-\xi s} ds\right]$$

$$= r - \ell(a^\star) + \lambda - \lambda \left[1 - \left(\frac{a^\star}{\bar{a}}\right)^{-\xi}\right] - \lambda \frac{\xi}{\xi - \gamma} \left(\frac{a^\star}{\bar{a}}\right)^{-\xi}. \tag{B.86}$$

Substituting for $\delta(a^\star)$ into (B.85) and setting $p(a^\star) = 1$, we obtain

$$e(a^\star) + p(a^\star) = \frac{\ell(a^\star) + \frac{\lambda\xi}{\xi-\gamma}\left(\frac{a^\star}{\bar{a}}\right)^{-\xi}}{r + \frac{\lambda}{\xi+1} - \mu + \frac{\lambda\xi}{\xi+1}\left(\frac{a^\star}{\bar{a}}\right)^{-(\xi+1)}}.$$

Simple computations show that this equation is equivalent to (22) if (23) holds, which we show below.

We are left to derive the liability constraint (23). From Lemma 1, we have $e(a) = 0$ for all $a \in [0, \bar{a}]$ and from Lemma 2, $e(a)$ strictly increases with a for $a \in [\bar{a}, \infty)$. Hence, limited liability holds for all a if $e(\bar{a}) = 0$. Using equation (17) with $\varphi = 0$ and $p(a^\star) = 1$, this condition writes

$$\left[e(a^\star) + p(a^\star)\right] \frac{\bar{a}}{a^\star} - 1 = 0, \tag{B.87}$$

which is equivalent to (23). This concludes the proof.

B.6 Proof of Proposition 4

We first show that $\bar{a} = 0$ when $\varphi = 1$. This result follows from equation (17) in Lemma 2. Setting $\varphi = 1$ and $p(a^\star) = 1$, it is clear that $e(\bar{a}) \geq 0$ for all $a \geq \bar{a}$ if $e(a^\star) \geq 0$. This latter condition is verified later in the existence result of Proposition 5.

For the second part of the proof, we rewrite equation (B.77) with $\bar{a} = 0$ to obtain

$$e(a^\star) = e(a^\star) + p(a^\star) - 1 = \frac{\mu^k - \delta(a^\star)}{r - \mu + \frac{\lambda}{\xi+1}}. \tag{B.88}$$

Substituting for $\delta(a^\star)$ thanks to equation (B.86), which becomes $\delta(a^\star) = r - \ell(a^\star)$ in this case, we obtain equation (24). This concludes the proof.

B.7 Proof of Proposition 5

Consider first the case $\varphi = 0$. Proposition 3 shows that an equilibrium with positive stablecoin value exists if there exist $(\bar{a}, a^\star)$ with $\bar{a} \leq a^\star$ such that condition (23) holds. Using equation (22) to substitute for $e(a^\star) + p(a^\star)$, this condition holds if there exists $a^\star$ and $x \in [0, 1]$ such that

$$\ell(a^\star)x - u - v(\gamma)x^{\xi+1} \geq 0, \quad \text{with} \quad u \equiv r + \frac{\lambda}{\xi+1} - \mu, \quad v(\gamma) \equiv \frac{\lambda\xi}{\xi+1} - \frac{\lambda\xi}{\xi-\gamma}. \tag{B.89}$$

To derive implications from this condition, define $H : x \mapsto \frac{x}{u+v(\gamma)x^{\xi+1}}$ and let x_{max} be the argument of the global maximum of H on $[0, 1]$. We have

$$H'(x) \propto u - v(\gamma)\xi x^{\xi+1},$$

which is strictly decreasing with x because $v(\gamma) > 0$ since $\gamma < -1$. Two cases are then possible. Either $H'(1) = u - \xi v(\gamma) \geq 0$ and $x_{max} = 1$ or $H'(1) < 0$ and $x_{max} = \left(\frac{u}{v(\gamma)\xi}\right)^{\frac{1}{\xi+1}}$ so that overall $x_{max} = \min\left\{1, \frac{u}{v(\gamma)\xi}\right\}^{\frac{1}{\xi+1}}$ and, for a given $a^\star$, a necessary condition for the desired equilibrium to exist is

$$\ell(a^\star) \geq \frac{u + v(\gamma)x_{max}^{\xi+1}}{x_{max}} = \frac{u + v(\gamma)\min\left\{1, \frac{u}{v(\gamma)\xi}\right\}}{\min\left\{1, \frac{u}{v(\gamma)\xi}\right\}^{\frac{1}{\xi+1}}} \tag{B.90}$$

A necessary condition for (B.90) to hold is $\ell(a^\star) \geq u$, as stated in Proposition 5.

Consider now case $\varphi = 1$. According to Proposition 4, a solution exists if there exists $a^\star$ such that $\ell(a^\star) + \mu^k - r \geq 0$. Given that $\max_a \ell(a) \geq \ell(a^\star)$ by definition, this condition can hold only if (25) holds. This concludes the proof.

B.8 Proof of Proposition 6

We first state a series of Lemmas and prove them at the end of this section.

Lemma 5. *The equity value $e(a)$ is weakly convex and continuously differentiable, and stablecoin price function $p(a)$ is continuous and increasing.*

Lemma 6. *If the equity value $e(a)$ is linear over some interval $[a_L, a_U]$, the equilibrium issuance policy features a target demand ratio $a^{jump} \in [a_L, a_U]$ such that the issuance policy for any $a \in [a_L, a_U]$ is to jump at a^{jump}.*

Lemma 7. *If $e(a)$ is strictly convex over some interval $[a_L, a_U]$, the equilibrium debt policy is smooth in that region. Furthermore, there is no MPE with positive stablecoin price if the equilibrium issuance policy is smooth everywhere.*

Proposition 6 is then a corollary of the next result.

Lemma 8. *If the interest rate policy is optimally chosen at date 0, there exists $(\bar{a}, a^{\star})$ such that the equilibrium issuance policy is smooth over $[0, \bar{a}]$ and features a jump at $a^{\star}$ when $a \in [\bar{a}, \infty)$.*

We now provide a proof for these lemmas.

Proof of Lemma 5. These properties follow from Lemma A.1 in DeMarzo and He (2021).
$\square$

Proof of Lemma 6. We first show that if the equity value $e(a)$ is linearly increasing in a over some segment $[a_L, a_U]$ (with strictly positive slope), the equilibrium issuance policy cannot be smooth over this interval. We then show that for any such interval $[a_L, a_U]$, there is a single jump point.

The proof is by contradiction. Suppose $d\mathcal{G}_t = G(a)dt$ over $[a_L, a_U]$ with $g(a) \equiv G(a)/C$, the stablecoin issuance rate per unit of stablecoins. With a smooth debt policy, use equation (B.65) to rewrite the HJB equation that governs stablecoin issuance as follows:

$$(r + \lambda)e(a) = \max_{g(a)} \left\{ \left(g(a)(p(a) - \varphi) + \mu a e'(a) + (\mu^k - \delta(a))\varphi \right. \right.$$
$$\left. \left. + (g(a) + \delta(a))(e(a) - e'(a)a) + \frac{\sigma^2}{2}a^2 e''(a) + \lambda \mathbb{E}[e(Sa)] \right\} \right(\quad \text{(B.91)}$$

A smooth debt policy is optimal if the first-order condition with respect to g is satisfied; that is, if

$$p(a) - \varphi = e'(a)a - e(a). \tag{B.92}$$

The assumption that $e(a)$ is linear in a further implies that $p'(a) = e''(a)a = 0$ and we denote $p(a) = p$ in what follows. Hence, equation (B.91) simplifies to

$$(r + \lambda)e(a) = \mu^k \varphi - \delta(a)p + \mu a e'(a) + \lambda \mathbb{E}[e(Sa)]. \tag{B.93}$$

We now establish a contradiction between equations (B.92) and (B.93) when $e(a)$ is linear. Taking the first-order-derivative with respect to a of the terms in (B.93), we obtain

$$(r + \lambda)e'(a) = -\delta'(a)p + \mu e'(a) + \lambda \mathbb{E}[e'(Sa)S]. \tag{B.94}$$

The HJB equation for the stablecoin price is given by

$$(r + \lambda)p(a) = \ell(a)p(a) + \delta(a)p(a) - (g(a) + \delta(a))ap'(a) + \mu a p'(a) + \frac{\sigma^2}{2}a^2 p''(a) + \lambda \mathbb{E}[p(Sa)], \tag{B.95}$$

which, for a constant $p(a) = p$, simplifies to

$$(r + \lambda)p = \ell(a)p + \delta(a)p + \lambda \mathbb{E}[p(Sa)]. \tag{B.96}$$

Combining equations (B.93), (B.94), and (B.96), we obtain

$$
\begin{align}
0 =& (r + \lambda)(p(a) - \varphi + e(a) - e'(a)a) \tag{B.97} \\
=& \ell(a)p + \delta(a)p + \lambda \mathbb{E}[p(Sa)] - (r + \lambda)\varphi + \mu^k \varphi - \delta(a)p + \mu a e'(a) \\
& + \lambda \mathbb{E}[e(Sa)] + \delta'(a)ap - \mu a e'(a) - \lambda \mathbb{E}[e'(Sa)Sa], \tag{B.98} \\
=& (\mu^k - r)\varphi + \ell(a)p + \delta'(a)ap(a) + \lambda \mathbb{E}[p(Sa) - \varphi + e(Sa) - e'(Sa)Sa], \tag{B.99} \\
=& (\mu^k - r)\varphi + \ell(a)p + \delta'(a)ap(a). \tag{B.100}
\end{align}
$$

The last equality follows from equation (B.92). We proved this relationship for segments in which the equilibrium issuance policy is smooth. For segments over which the issuance

policy features jumps, equation (20) shows that for any a, a' in this segment, we have

$$e(a') = \left[e(a) + p - \varphi\right]\frac{a'}{a} - (p - \varphi). \tag{B.101}$$

Taking the first-order derivative with respect to a' and then setting $a' = a$, we obtain equation (B.92).

We now establish a contradiction. Suppose first that $a_L = 0$. Thus $p = \varphi$ and $\mathbb{E}[p'(Sa)S] = 0$ for $a \in [a_L, a_U]$. Then, if $p \neq 1$ and thus $\ell(a) = 0$, it is immediate that equations (B.100) and (B.96) are inconsistent. If instead $p = 1$, these two equations imply that

$$(\mu^k - r)\varphi + \ell(a) - \ell'(a)a = 0,$$

but the functional form of $\ell(a)$ cannot be pinned down by these equilibrium conditions because it is a primitive of the problem.

Suppose now that $a_L > 0$. Suppose first that $p(a) = p \neq 1$, in which case $\ell(a) = 0$ by definition. Equations (B.100) and (B.96) then imply that

$$\delta'(a) = \frac{r - \mu^k}{ap}\varphi = -\lambda\mathbb{E}[p'(Sa)S]. \tag{B.102}$$

This equation cannot hold, because $r > \mu^k$ while $p' \geq 0$ by Lemma 5. Finally, suppose that $p(a) = 1$. Equations (B.96) and (B.100) imply together that

$$\frac{(\mu^k - r)\varphi + \ell(a) - \ell'(a)a}{a} = \lambda\mathbb{E}[p'(Sa)S]. \tag{B.103}$$

We have

$$\mathbb{E}[p'(Sa)S] = \int_0^\infty p'(e^{-s}a)\xi e^{-s(\xi+1)}ds \tag{B.104}$$

$$= \int_{\ln(a/a_L)}^\infty p'(e^{-s}a)\xi e^{-s(\xi+1)}ds = \kappa a^{-(\xi+1)} \tag{B.105}$$

where $\kappa \equiv a_L{}^{\xi+1}\int_0^\infty p'(e^{-s}a_L)\xi e^{-s(\xi+1)}ds$ is a positive constant. To obtain the second line, we use the fact that p is constant over $[a_L, a_U]$. Thus, we must have

$$\ell(a) = \ell'(a)a - (\mu^k - r)\varphi a + \lambda\kappa a^{-\xi} \tag{B.106}$$

68

for $a \in [a_L, a_U]$. A general solution to this equation is of the form

$$\ell(a) = \alpha a + \beta + f a^{-\xi - 1},$$

with $f \geq 0$. Hence, assuming that the issuance policy is smooth imposes a functional form for $\ell(a)$. This leads to a contradiction because $\ell(a)$ is an exogenous function in this problem.

We now show that there can only be one jump point $a_{jump} \in [a_L, a_U]$ if $e(a)$ is linear over $[a_L, a_U]$. Suppose there are two such jump points (the argument generalizes for more jump points) labeled a^1_{jump} and a^{jump}_2. Then, the single-peak property in Assumption 1 ensures that there must be one jump point—say, a^1_{jump}—for which liquidity benefits $\ell(a)/a * A$ are larger than at a^2_{jump}. Hence, to maximize its date-0 value, the platform would strictly prefer jumping to a^1_{jump} from any point in $[a_L, a_U]$ rather than to a^2_{jump}.

We are left to show that jumping to a^1_{jump} instead of a^2_{jump} is compatible with the equilibrium issuance policy. By Lemmas 7 and 6, the issuance policy features jumps on $[a_L, a_U]$ only if equity value is linear and price is constant. Hence, from any state a with jump point a^2_{jump}, we have

$$e(a) = \left[e(a^2_{jump}) + p(a^2_{jump}) - \varphi \right] \frac{a}{a^2_{jump}} = \left[e(a^1_{jump}) + p(a^1_{jump}) - \varphi \right] \frac{a}{a^1_{jump}}.$$

Hence, jumping to a^1_{jump} is also an optimal equilibrium issuance policy. This equality simply reflects the fact that the platform is indifferent ex post between all points in $[a_L, a_U]$. At date-0, however, the platform would choose jump point a^1_{jump} as the sole jump point. □

Proof of Lemma 7. We first show that if the equity value is strictly convex in C over some interval, the issuance policy is smooth in this region. Given any debt level $\hat{C}$, equity holders have the option to adjust the stock of stablecoins to C by issuing $C - \hat{C}$ at the price of $p(A, C)$. Therefore, by optimality of the debt issuance policy, the equity value at $\hat{C}$ must satisfy

$$E(A, \hat{C}) \geq E(A, C) + p(A, C)(C - \hat{C}). \tag{B.107}$$

To show that discrete repurchases are suboptimal, we prove that inequality (B.107) is strict if the equity value is strictly convex with respect to its second argument. Suppose, to the

contrary, that there exists $C' \neq C$ such that $E(A, C') = E(A, C) + p(A, C)(C - C')$. By strict convexity of E, we get that for all $x \in]0, 1[$

$$E(A, xC + (1 - x)C') < xE(A, C) + (1 - x)E(A, C') = E(A, C) + (1 - x)p(A, C)(C - C').$$
$$(B.108)$$

Using then condition (B.107) for $\hat{C} = xC + (1 - x)C'$, we obtain

$$E(A, xC + (1 - x)C') \geq E(A, C) + (1 - x)p(A, C)(C - C'), \qquad (B.109)$$

which is a contradiction with (B.108). Thus, it must be that

$$E(A, C') > E(A, C) + p(A, C)(C - C'). \qquad (B.110)$$

Hence, any discrete issuance with $|C - C'| > 0$ would be suboptimal for shareholders; that is, the debt policy must be smooth if E is strictly convex in C.

Second, we show that there cannot be an equilibrium with positive platform value and a smooth debt policy for all a. For the equilibrium issuance policy to be smooth, it must be that equation (B.92) holds. The platform starts at date 0 if liquidity benefits can be captured in equilibrium. Two cases are possible, given that p is weakly increasing with a. First, there exists an interval $[a_L, a_U]$ over which the price is constant with $p(a) = 1$. Equation (B.92) then implies that e is linear. We can then use Lemma 6 to show that the equilibrium debt policy features jump, a contradiction. The second case is that of a single point $\hat{a}$ for which $p(\hat{a}) = 1$ and such that the platform spends strictly positive time at $\hat{a}$. Such a feature requires that the platform perform a control at $\hat{a}$. The same arguments used in DeMarzo and He (2021), however, show that such a policy cannot be part of an equilibrium in a region in which the equity value is strictly convex.

$\square$

Proof of Lemma 8. From Lemma 5, we know that since the equity value $e(a)$ is weakly convex, there must be a strictly ordered sequence $\{a^{(n)}\}_{n \geq 0}$ such that $a_0 = \underline{a}$ is the default threshold and $\lim_{n \to \infty} a^{(n)} = \infty$ such that on each segment $[a^{(n)}, a^{(n+1)}]$, e is either strictly convex or linear, with different convexity on two consecutive segments.

Our second step is to show that there is at least 1 segment with $e(a)$ strictly convex

(possibly empty), and one segment with $e(a)$ linear. We first establish that the equity value cannot be linear on segment $[a^{(0)}, a^{(1)}]$ unless $a^{(0)} = 0$ and $\varphi = 1$. Suppose first that $a^{(0)} > 0$ so that the platform may default in equilibrium. If $e(a)$ is linear over $[a^{(0)}, a^{(1)}]$, there is a kink in the equity value at $a^{(0)}$ such that $\lim_{a \downarrow a^{(0)}} e'(a) \neq 0$, which is incompatible with an optimal default decision and the corresponding smooth-pasting condition. Suppose now that $a^{(0)} = 0$ so that the platform never defaults in equilibrium. If $e(a)$ is linear on $[0, a^{(1)}]$, there must be $a^{jump} \in [0, a^{(1)}]$ such that the issuance policy is to jump at a^{jump} from any point in $[0, a^{(1)}]$ by Lemma 6. This implies that for any $a \in [0, a^{(1)}]$

$$
e(a) = \left[e(a^{jump}) + p(a^{jump}) - \varphi\right]\frac{a}{a^{jump}} - (p(a^{jump}) - \varphi),
$$

with $p(a^{jump})$ constant over $[0, a^{(1)}]$ and $p(a^{jump}) > \varphi$ unless $\varphi = 1$. Hence, when $a \to 0$ limited liability is violated, except in the case $\varphi = 1$. This proves that the equity value is strictly convex over $[0, a^{(1)}]$ unless $\varphi = 1$ and $\underline{a} = 0$. In that case, the equilibrium equity value may be linear for all a.

Second, Lemma 7 implies that there must exist a segment over which $e(a)$ is linear. The last step of the proof is to show that there exists $\bar{a}$ such that the equity value is strictly convex over $[\underline{a}, \bar{a}]$ and linear over $[\bar{a}, \infty)$. Characterization of the equilibrium issuance policy as a targeted Markov policy then follows from Lemmas 5, 6, and 7. Let $\delta(a)$ be an interest policy that induces a nonzero MPE with issuance policy $d\mathcal{G}$ such that there exists a segment $[a^{(2)}, a^{(3)}]$ over which e is strictly convex; call it the original (interest rate) policy for short. We want to show that there exists an alternative interest rate policy $\hat{\delta}(a)$ that induces a Markov equilibrium with issuance policy $d\hat{\mathcal{G}}$ such that $e(a)$ has the desired properties and the date-0 platform value is strictly higher.

We first construct an alternative policy and its induced equilibrium. Let $a^\star$ be the target value in the first linear region $[a^{(1)}, a^{(2)}]$ for equity in the equilibrium induced by the original policy. Construct the alternative policy and the induced equilibrium as follows. Set $\hat{\delta}(a) = \delta(a)$ for all a and $d\hat{\mathcal{G}}(a, C) = d\mathcal{G}(a, C)$ for $a \leq a^\star$ and $d\hat{\mathcal{G}}(a, C) = A/a^\star - C$ for $a \geq a^\star$. Next, set the same default policy $\hat{\underline{a}} = \underline{a}$. Finally, conjecture that in the equilibrium induced by the alternative policy, the equity value $\hat{e}(a)$ is linear and the price $\hat{p}(a)$ is constant for all $a \in [a^{(1)}, \infty)$.

Next, we argue that the issuance policy $d\hat{\mathcal{G}}(a, C)$ and the default policy $\hat{\underline{a}}$ are equilibrium policies induced by the alternative interest rate policy $\hat{\delta}(a)$. The subspace $[0, a^\star]$ is absorb-

71

ing for the equilibrium induced by the original policy, because there are only downward jumps to A and the platform jumps to $a^\star$ from any $a \in [a^{(1)}, a^{(2)}]$. Hence, the fact that $d\mathcal{G}(a, C)$ for $a \in [0, a^{(2)}]$ is an equilibrium issuance policy induced by the original interest rate policy implies that $d\hat{\mathcal{G}}(a, C)$ for $a \in [0, a^{(2)}]$ is an equilibrium issuance policy induced by the alternative interest rate policy. The same argument applies to the default threshold $\underline{\hat{a}} = \underline{a}$. This argument also implies that $\hat{e}(a) = e(a)$ and $\hat{p}(a) = p(a)$ for all $a \in [0, a^\star]$. We are thus left to show that $d\hat{\mathcal{G}}(a, C)$ is an equilibrium issuance policy on the rest of the state space, $a \in [a^{(2)}, \infty)$. This result follows from the observation that $\hat{e}(a)$ is linear over $a \in [a^{(1)}, \infty)$ and $\hat{p}(a)$ is constant. This implies that jumping to any point in $a \in [a^{(1)}, \infty)$, including a^*, can be part of an equilibrium issuance policy, as shown above.

Third, we show that $p(a) = 1$ for $a \in [a^{(1)}, a^{(2)}]$ in the equilibrium induced by the original policy, and thus $\hat{p}(a) = 1$ for all $a \in [a^{(1)}, \infty)$. Equity value is linear over $[a^{(1)}, a^{(2)}]$ and the equilibrium issuance policy is to jump at $a^\star \in [a^{(1)}, a^{(2)}]$ when $a \in [a^{(1)}, a^{(2)}]$. Hence, the price $p(a) = p$ must be constant over $[a^{(1)}, a^{(2)}]$. Since $[0, a^\star]$ is an absorbing subspace for the equilibrium induced by the original policy, it must be that $p = 1$. If not, investors never enjoy any liquidity benefit for $a \in [0, a^\star]$ and thus $p(a) = e(a) = 0$ for all $a \in [0, a^\star]$, which is a contradiction. To see this, suppose first that $p < 1$. By monotonicity of p, we have $p(a) < 1$ for all $a \in [0, a^{(2)}]$, which implies that investors never enjoy the liquidity benefit. Conversely, if $p > 1$ over $[a^{(1)}, a^{(2)}]$, we have $p(a) = 1$ for a unique $a \in [0, a^{(1)})$ because $p(a)$ is strictly increasing over $[0, a^{(1)})$, since $e(a)$ is strictly convex (see the proof of Lemma 7). With a smooth equilibrium issuance policy on $[0, a^{(1)}]$, this state is not visited with positive probability and thus investors enjoy liquidity benefit with zero probability, which again leads to a contradiction. Hence $p(a) = 1$ for $a \in [a^{(1)}, a^{(2)}]$. This implies $\hat{p}(a) = 1$ for all $a \in [a^{(1)}, \infty)$ in the equilibrium induced by the alternative policy.

Finally, we can show that the platform value at date 0 is higher under the alternative policy than under the original policy. The platform's value at date 0 is given by equation (10), which we rewrite here for convenience.

$$E_0 = \mathbb{E}\left[\iint\left(e^{-rt}\ell(A_t, C_t)C_t \mathbf{1}_{p(A_t, C_t)=1} + (\mu^k - r)\varphi C dt \mid A_0, C_0 = 0 \right]\right(\tag{B.111}$$

In any equilibrium, liquidity benefits are only enjoyed when $a \in [a^{(1)}, a^{(2)}]$ because $p(a) = 1$ for $a \in [a^{(1)}, a^{(2)}]$. Under the alternative policy, $a^\star \in [a^{(1)}, a^{(2)}]$ is reached immediately at date 0 by design because the equilibrium issuance policy is to jump to $a^\star$ when no

stablecoins are outstanding ($a = \infty$). In the equilibrium induced by the original policy, however, the optimal choice at date 0 is some $a^{\star\star} > a^{(2)}$ by design of the original policy. Denote τ_f the first (stochastic) time the platform enters the region $[a^{(1)}, a^{(2)}]$ under the original policy. We have

$$E_0 = \mathbb{E}[E^{-r\tau_f}]\hat{E}_0 + \mathbb{E}\left[\iint_0^\infty e^{-rt}(\mu^k - r)\varphi C dt\right] < E_0, \tag{B.112}$$

because no liquidity benefit is enjoyed before the platform reaches $[a^{(1)}, a^{(2)}]$. The inequality follows from the fact that $\mathbb{E}[\tau_f] > 0$ by design of the original policy and $\mu^k < r$.

We have shown that the original policy is strictly dominated. Hence, in an equilibrium induced by an optimal interest rate policy, the issuance policy must belong to the class of targeted Markov policies. □

This concludes the proof of Proposition 6.

B.9 Proof of Proposition 7

Point 1. We derive the equilibrium stablecoin issuance rate in the smooth region $[\underline{a}, \overline{a}]$. Our analysis in the proof of Proposition 6 shows that a smooth debt issuance policy is optimal if and only if equation (B.92) holds. We will solve for the equilibrium value of g thanks to this equation. Taking the first-order derivative of e in equation (B.91) at $g = 0$, we obtain

$$(r + \lambda)e'(a) = \mu(e'(a) + ae''(a)) - \delta'(a)p(a) - p'(a)\delta(a) + \frac{\sigma^2}{2}a(2e''(a) + ae'''(a)) + \lambda\mathbb{E}[Se'(Sa)] \tag{B.113}$$

The HJB for the stablecoin price is given by equation (B.95) with $\ell(a) = 0$, because the price p is strictly below one by construction in the smooth region. We can then use (B.92)

to obtain a condition on g. We have

$$0 = (r + \lambda)(p(a) - \varphi + e(a) - e'(a)a) \tag{B.114}$$

$$= \delta(a)p(a) - (g(a) + \delta(a))p'(a)a + \mu a p'(a) + \frac{\sigma^2}{2}a^2 p''(a) + \lambda \mathbb{E}[p(Sa)] - (r + \lambda)\varphi \tag{B.115}$$

$$+ \mu^k \varphi - \delta(a)p(a) + \mu a e'(a) + \frac{\sigma^2}{2}a^2 e''(a) + \lambda \mathbb{E}[e(Sa)] \tag{B.116}$$

$$+ \delta(a)p'(a)a + \delta'(a)p(a)a - \mu a^2 e''(a) - \mu a e'(a) - \frac{\sigma^2}{2}a^3 e'''(a) - \sigma^2 a^2 e''(a) - \lambda \mathbb{E}[e'(Sa)Sa] \tag{B.117}$$

$$= (\mu^k - r)\varphi - g(a)a p'(a) + \delta'(a)a p(a) + \mu a \underbrace{(p'(a) - a e''(a))}_{=0} + \frac{\sigma^2}{2}a^2 \underbrace{(p''(a) + e''(a) - a e'''(a))}_{=0} \tag{B.118}$$

$$+ \lambda \mathbb{E}[\underbrace{p(Sa) - \varphi + e(Sa) - e'(Sa)Sa}_{=0}] \tag{B.119}$$

$$= (\mu^k - r)\varphi - g(a)a p'(a) + \delta'(a)a p(a). \tag{B.120}$$

To obtain the last equation, we use (B.92) to set the last term to 0. Differentiating equation (B.92) further shows

$$p'(a) = e''(a)a, \tag{B.121}$$

$$p''(a) = e'''(a)a + e''(a), \tag{B.122}$$

which allow us to set other terms to 0. This proves our claim.

Point 2. Next, we derive the smooth-pasting condition at $\underline{a}$ and $\bar{a}$. First, consider default threshold $\underline{a}$ if $\underline{a} > 0$. Equation (7) shows that equity value in default is equal to 0, because the collateralization rate φ is lower than 1 by assumption. Hence, the default threshold is chosen optimally if and only if condition (30) holds. Consider now the lower bound of the target region $\bar{a}$. For $a \geq \bar{a}$, the equity value is given by (B.128). Hence, we obtain for all $a = a/C \geq \bar{a}$

$$e'(a) = E_A(A, C) = \frac{e(a^\star) + 1 - \varphi}{a^\star}. \tag{B.123}$$

Hence, continuity of the derivative of e with respect to a at $\bar{a}$ implies condition (29).

74

Point 3. Finally, we derive conditions to rule out ex post deviations from the conjectured equilibrium policy in the target region $[\bar{a}, \infty)$. The conjectured issuance policy features a jump to $a^\star$ from any point in the target region. To derive conditions for this policy to be ex post optimal, we consider "one-step" deviations whereby the platform deviates and then follows the equilibrium policy from the value of the demand ratio following the deviation.

We first show that we only need to consider smooth deviations. Proposition 6 shows that in the target region, the equilibrium equity value must be given by

$$E(A, C) = E(A, C^\star(A)) + (p(a^\star) - \varphi)(C^\star(A) - C), \tag{B.124}$$

with $p(A, C) = p(a^\star)$ for any $a = A/C \geq \bar{a}$. The value when jumping to a ratio $\hat{a} \in [\bar{a}, \infty)$ is thus

$$\hat{E}(A, C) = E(A, \hat{C}(A)) + (p(a^\star) - \varphi)(\hat{C}(A) - C) = E(A, C^\star(A)) + (p(a^\star) - \varphi)(C^\star(A) - C), \tag{B.125}$$

with $\hat{C}(A) \equiv A/\hat{a}$. Hence, from $a \in [\bar{a}, \infty)$, jumping to $\hat{a}$ gives the platform the same utility as the equilibrium policy. The platform cannot gain from jumping to a different point of the target region because it then jumps instantaneously to target demand ratio $a^\star$. Next, a jump to some ratio $\hat{a} \leq \bar{a}$ can be ruled out because the equity value is strictly convex in C for $a \in [\underline{a}, \bar{a}]$. The value from such a jump is indeed

$$\hat{E}(A, C) = E(A, \hat{C}(A)) + (p(\hat{a}) - \varphi)(\hat{C}(A) - C) < E(A, C) \tag{B.126}$$

We are thus left to derive conditions such that for any $a \in [\bar{a}, \infty)$, a smooth issuance policy deviation is suboptimal under condition (31). Given that the return to issuance is zero by construction, it is enough to check that equity owners prefer the equilibrium policy over inaction during time interval dt. For state (A, C) with $A/C \geq \bar{a}$, the equilibrium value of equity is given by (B.124). If, instead, equity owners stay inactive during time interval dt before reverting to the equilibrium policy, they enjoy

$$\hat{E}(A, C) = \mu^k \varphi C dt - \varphi \delta(a) C dt + (1 - rdt)(1 - \lambda dt) \mathbb{E}\left[E(A + dA, C + \delta(a)Cdt)\right]$$
$$+ (1 - rdt)\lambda dt \mathbb{E}[E(SA, C)]. \tag{B.127}$$

When $a \in [\underline{a}, \infty)$, rewriting (B.124) the equilibrium equity value is given by

$$E(A, C) = \frac{A}{a^\star} e(a^\star) + (p(a^\star) - \varphi)(C^\star(A) - C) = \frac{e(a^\star) + p(a^\star) - \varphi}{a^\star} A - (p(a^\star) - \varphi)C. \quad \text{(B.128)}$$

Hence, we get

$$\mathbb{E}\left[E(A + dA, C + \delta(a)Cdt)\right] = E(A, C) + \mu \frac{e(a^\star) + p(a^\star) - \varphi}{a^\star} A dt - (p(a^\star) - \varphi)\delta(a)Cdt. \quad \text{(B.129)}$$

Plugging (B.129) into (B.127) and keeping only terms of order at least dt, we obtain

$$\hat{E}(A, C) = E(A, C) - (r + \lambda)E(A, C)dt + \mu\left[e(a^\star) + p(a^\star) - \varphi\right]C^\star(A)dt$$
$$- p(a^\star)\delta(a)Cdt + \mu^k \varphi Cdt + \lambda\mathbb{E}[E(SA, C)]dt. \quad \text{(B.130)}$$

Equity owners do not deviate if and only if $\hat{E}(A, C) < E(A, C)$; that is, if

$$(r + \lambda)E(A, C) \geq \mu\left[e(a^\star) + p(a^\star) - \varphi\right]C^\star(A) - p(a^\star)\delta(a)C + \mu^k \varphi C + \lambda\mathbb{E}[E(SA, C)], \quad \text{(B.131)}$$

which is equivalent to

$$(r + \lambda)\left[e(a^\star)C^\star(A) + (p(a^\star) - \varphi)(C^*(A) - C)\right] \geq \mu\left[e(a^\star) + p(a^\star) - \varphi\right]C^\star(A) - p(a^\star)\delta(a)C$$
$$+ \mu^k \varphi C + \lambda\mathbb{E}[E(SA, C)], \quad \text{(B.132)}$$

where we used equation (B.128) to substitute for $E(A, C)$. Rearranging terms, (B.132) can be written as

$$(r + \lambda - \mu)e(a^\star)C^\star(A) \geq -(r + \lambda)(p(a^\star) - \varphi)(C^*(A) - C) + \mu(p(a^\star) - \varphi)C^\star(A)$$
$$- p(a^\star)\delta(a)C + \mu^k \varphi C + \lambda\mathbb{E}[E(SA, C)]. \quad \text{(B.133)}$$

Using now equation (18) to substitute for $e(a^\star)$, we get

$$\mu^k \varphi C^\star(A) - p(a^\star)\delta(a^\star)C^\star(A) + \mu(p(a^\star) - \varphi)C^\star(A) + \lambda\mathbb{E}[E(SA, C^\star(A))] - \lambda\mathbb{E}[E(SA, C)] \geq$$
$$- (r + \lambda)(p(a^\star) - \varphi)(C^*(A) - C) + \mu(p(a^\star) - \varphi)C^\star(A) - p(a^\star)\delta(a)C + \mu^k \varphi C, \quad \text{(B.134)}$$

which we can finally rewrite as

$$\left[(r+\lambda)(p(a^\star)-\varphi)+\mu^k\varphi\right](C-C^\star(A)) \leq \lambda\mathbb{E}[E(SA, C^\star(A))] - \lambda\mathbb{E}[E(SA, C)]$$
$$+ p(a^\star)(\delta(a)C - \delta(a^\star)C^\star(A)). \qquad \text{(B.135)}$$

Setting $p(a^\star) = 1$ and dividing all terms by C, (B.135) is equivalent to (31). This concludes the proof.

B.10   Equity and Price Characterization with Limited Commitment

We first verify our guess for the equity value and the price function. Using HJB equation (B.91) together with condition (32) and Assumption 4, we obtain the following HJB equation for the equity value:

$$(r+\lambda)e(a) = (\mu^k - \underline{\delta})\varphi + \underline{\delta}(e(a) - ae'(a)) + \mu ae'(a) + \frac{\sigma^2}{2}a^2 e''(a) + \lambda\mathbb{E}[e(Sa)]. \qquad \text{(B.136)}$$

Then, we compute the term $\mathbb{E}[e(Sa)]$ using the conjectured $e(a)$. We have

$$\mathbb{E}[e(Sa)] = \int_0^\infty \left\{ e(e^{-s}a)\xi e^{-\xi s} \right\} ds = \int_0^{\ln(a/\underline{a})} \left[\underline{e} + \left\{ \sum_{k=1}^{3} c_k e^{s\gamma_k} a^{-\gamma_k} \right\} \right] \xi e^{-\xi s} ds$$
$$= \underline{e}\left(1 - \left(\frac{a}{\underline{a}}\right)^{-\xi}\right) + \sum_{k=1}^{3} \frac{c_k\xi}{\xi - \gamma_k} a^{-\gamma_k}\left(1 - \left(\frac{a}{\underline{a}}\right)^{-(\xi-\gamma_k)}\right). \qquad \text{(B.137)}$$

We then plug in guess (38) into the HJB to obtain

$$(r+\lambda-\underline{\delta})\left[\underline{e} + \sum_{k=1}^{3} c_k a^{-\gamma_k}\right] = (\mu^k - \underline{\delta})\varphi - (\mu - \underline{\delta})\sum_{k=1}^{3} \gamma_k c_k a^{-\gamma_k}$$
$$+ \frac{\sigma^2}{2}\sum_{k=1}^{3}(1 + \gamma_k)\gamma_k c_k a^{-\gamma_k} + \lambda\mathbb{E}[e(Sa)]. \qquad \text{(B.138)}$$

Several conditions are necessary for this equation to hold. Equating first constant terms on each side of (B.138), we can solve for $\underline{e}$:

$$\underline{e} = \frac{\mu^k - \underline{\delta}}{r - \underline{\delta}}\varphi. \qquad \text{(B.139)}$$

77

Next, since the terms in $a^{-\gamma_k}$ must be equal on each side of (B.138), γ_k must solve equation (40) for $k \in \{1, 2, 3\}$. The roots of that polynomial are given by

$$\gamma_k = -\frac{1}{2t_1}\left(t_2 + \zeta^k R + \frac{\Delta_0}{\zeta^k R}\right)\Big($$
(B.140)

where

$$\Delta_0 = t_2^2 - 3t_1 t_3, \quad \Delta_1 = 2t_2^3 - 9t_1 t_2 t_3 + 27t_1^2 t_4,$$

$$R = \sqrt[3]{\frac{\Delta_1 + \sqrt{\Delta_1^2 - 4\Delta_0^3}}{2}}, \quad \zeta = \frac{-1 + \sqrt{-3}}{2}, \quad k = \{0, 1, 2\},$$

$$t_1 = -\frac{\sigma^2}{2}, \quad t_2 = \mu - \underline{\delta} + \frac{\sigma^2}{2}(\xi - 1), \quad t_3 = -(\mu - \underline{\delta})\xi + \frac{\sigma^2}{2}\xi + r - \underline{\delta} + \lambda, \quad t_4 = -(r - \underline{\delta})\xi,$$

where γ_k is decreasing with k. According to Descartes' rule of sign, this polynomial has 2 positive roots and 1 negative root if $\underline{\delta} < r$, 1 positive root and 1 negative root if $\underline{\delta} = r$, and 1 positive root and 2 negative roots if $r < \underline{\delta}$. Using Budan-Fourier theorem, we can show that exactly one negative root is strictly lower than -1. Using the implicit function theorem and the Gauss-Lucas theorem, we can further show that $\partial\gamma_3/\partial\underline{\delta} < 0$.

To solve for the parameters c_k's, we use the matching conditions imposed by continuity of $e(\cdot)$ at $\underline{a}$ and $\bar{a}$ and the memoryless property of the exponential distribution of downward jumps. Continuity at $\underline{a}$ and $\bar{a}$ imply that

$$\underline{e} + \sum_{k=1}^{3} c_k \underline{a}^{-\gamma_k} = 0,$$
(B.141)

$$\underline{e} + \sum_{k=1}^{3} c_k \bar{a}^{-\gamma_k} = (e(a^\star) + p(a^\star) - \varphi)\frac{\bar{a}}{a^\star} - (p(a^\star) - \varphi).$$
(B.142)

Next, the terms in $a^{-\xi}$ on each side of (B.136) must cancel out:

$$\underline{e} + \sum_{k=1}^{3} \frac{c_k \xi}{\xi - \gamma_k}\underline{a}^{-\gamma_k} = 0,$$
(B.143)

which is equivalent to

$$\mathbb{E}[e(S\underline{a})] = 0.$$
(B.144)

That is, thanks to the memoryless property of the exponential distribution, we only need condition (B.143) to solve for the expected value of a downward jump below $\underline{a}$.

Finally, for $\underline{a}$ and $\bar{a}$ to be optimal, smooth-pasting conditions (29) and (30) from Proposition 7 must be satisfied:

$$-\sum_{k=1}^{3} c_k \gamma_k \bar{a}^{-(\gamma_k+1)} = \frac{e^\star + 1 - \varphi}{a^\star}, \tag{B.145}$$

$$-\sum_{k=1}^{3} c_k \gamma_k \underline{a}^{-(\gamma_k+1)} = 0. \tag{B.146}$$

We proceed similarly for the price function. Using $\delta(a) = \underline{\delta}$ by Assumption 4 and plugging the equilibrium issuance equation (28) into the HJB for the price, equation (B.95), we obtain

$$(r + \lambda - \underline{\delta})p(a) = (r - \mu^k)\varphi + (\mu - \underline{\delta})ap'(a) + \frac{\sigma^2}{2}a^2 p''(a) + \lambda \mathbb{E}[p(Sa)]. \tag{B.147}$$

Then, we compute the term $\mathbb{E}[p(Sa)]$ using the conjectured $p(a)$. We have

$$\begin{aligned}
\mathbb{E}[p(Sa)] &= \int_0^\infty \left\{ p(e^{-s}a)\xi e^{-\xi s} \right\} ds \\
&= \int_0^{\ln(a/\underline{a})} \left[\underline{p} + \left\{ \sum_{k=1}^{3} b_k e^{s\gamma_k} a^{-\gamma_k} \right\} \right] \xi e^{-\xi s} ds + \int_{\ln(a/\underline{a})}^\infty \varphi \xi e^{-\xi s} ds \\
&= \underline{p}\left(1 - \left(\frac{a}{\underline{a}}\right)^{-\xi}\right) + \sum_{k=1}^{3} \frac{b_k \xi}{\xi - \gamma_k} a^{-\gamma_k}\left(1 - \left(\frac{a}{\underline{a}}\right)^{-(\xi-\gamma_k)}\right) + \varphi\left(\frac{a}{\underline{a}}\right)^{-\xi}. \tag{B.148}
\end{aligned}$$

Equation (B.147) holds if the constant term $\underline{p}$ in (39) solves

$$\underline{p} = \frac{r - \mu^k}{r - \underline{\delta}}\varphi, \tag{B.149}$$

and if γ_k is a solution to equation (40) for $k \in \{1, 2, 3\}$. Next, the matching conditions

$p(\underline{a}) = \varphi$ and $p(\overline{a}) = 1$ imply, respectively,

$$\underline{p} + \sum_{k=1}^{3} b_k \underline{a}^{-\gamma_k} = \varphi \tag{B.150}$$

$$\underline{p} + \sum_{k=1}^{3} b_k \overline{a}^{-\gamma_k} = 1. \tag{B.151}$$

Finally, from the memoryless property of the exponential distribution, we get

$$\underline{p} + \sum_{k=1}^{3} \frac{b_k}{\xi - \gamma_k} \underline{a}^{-\gamma_k} = \varphi. \tag{B.152}$$

Next, we derive the platform's objective function as a function of parameters. From equations (18) and (19), we obtain

$$e(a^\star) + 1 - \varphi = \frac{\ell(a^\star) + (\mu^k - r)\varphi + \lambda \mathbb{E}[e(Sa^\star) + p(Sa^\star) - \varphi]}{r + \lambda - \mu}. \tag{B.153}$$

Now, we solve for $\mathbb{E}[e(Sa^\star)]$ and $\mathbb{E}[p(Sa^\star)]$ using the functional forms (38) and (39). We have

$$
\begin{aligned}
\mathbb{E}[e(Sa^\star)] &= \int_0^{\ln(a^\star/\overline{a})} \left[(e(a^\star) + 1 - \varphi)e^{-s} - (1 - \varphi) \right] \xi e^{-\xi s} ds \\
&+ \int_{\ln(a^\star/\overline{a})}^{\ln(a^\star/\underline{a})} \left[\underline{e} + \sum_{k=1}^{3} c_k (a^\star)^{-\gamma_k} e^{s\gamma_k} \right] \xi e^{-\xi s} ds \\
&= \frac{\xi}{\xi + 1} \left[1 - \left(\frac{a^\star}{\overline{a}} \right)^{-(\xi+1)} \right] \left(e(a^\star) + 1 - \varphi \right) - \left[1 - \left(\frac{a^\star}{\overline{a}} \right)^{-\xi} \right] (1 - \varphi) + \left[\left(\frac{a^\star}{\overline{a}} \right)^{-\xi} - \left(\frac{a^\star}{\underline{a}} \right)^{-\xi} \right] \underline{e} \\
&+ \sum_{k=1}^{3} \frac{\xi c_k}{\xi - \gamma_k} (a^\star)^{-\gamma_k} \left[\left(\frac{a^\star}{\overline{a}} \right)^{-(\xi-\gamma_k)} - \left(\frac{a^\star}{\underline{a}} \right)^{-(\xi-\gamma_k)} \right]
\end{aligned} \tag{B.154}
$$

Turning now to the price term, we have

$$\mathbb{E}[p(Sa^\star)] = \int_0^{\ln(a^\star/\bar{a})} \xi e^{-\xi s} ds + \int_{\ln(a^\star/\bar{a})}^{\ln(a^\star/\underline{a})} \left[\underline{p} + \sum_{k=1}^{3} b_k \, (a^\star)^{-\gamma_k} \, e^{s\gamma_k} \right] \xi e^{-\xi s} ds + \int_{\ln(a^\star/\underline{a})}^{\infty} \varphi \xi e^{-\xi s} ds$$

$$= 1 - \left(\frac{a^\star}{\bar{a}} \right)^{-\xi} + \left[\left(\frac{a^\star}{\bar{a}} \right)^{-\xi} - \left(\frac{a^\star}{\underline{a}} \right)^{-\xi} \right] \underline{p}$$

$$+ \sum_{k=1}^{3} \left(\frac{\xi b_k}{\xi - \gamma_k} (a^\star)^{-\gamma_k} \left[\left(\frac{a^\star}{\bar{a}} \right)^{-(\xi-\gamma_k)} - \left(\frac{a^\star}{\underline{a}} \right)^{-(\xi-\gamma_k)} \right] + \varphi \left(\frac{a^\star}{\underline{a}} \right)^{-\xi} \right). \quad \text{(B.155)}$$

Using equations (B.154) and (B.155), we can thus express the platform's objective in (B.153) as a function of the parameters $\{\underline{a}, \bar{a}, a^\star, \underline{\delta}\}$ of the TMP and the parameters of the functional forms for $e(a)$ and $p(a)$ in (38) and (39), which themselves depend on the TMP's parameters via equations (B.139), (B.141), (B.142), (B.143), (B.145), (B.146), (B.149), (B.150), (B.151), and (B.152).

B.11   Proof of Proposition 8

We first prove that the platform does not default; that is, $\underline{a} = 0$. From (B.139), the constant term $\underline{e}$ in the equity value function (38) is equal to 0 and is thus (weakly) positive for any value of $\underline{\delta}$. This implies that the option value to default has no value, so the default threshold is $\underline{a} = 0$.

Second, the fact that $\underline{a} = 0$ implies that the only relevant root of characteristic equation (40) is the root strictly below -1. Consider instead the conditions on the equity value function. Equation (B.142), which imposes continuity at 0, implies that γ_k must be negative. Smooth-pasting condition (B.146) further implies that $\gamma_k < -1$. The same conclusion applies to the price function, because (32) holds in the smooth region. For ease of notation, we now call γ this root and b and c the corresponding coefficients for the price function and the equity value function, respectively.

We now restate continuity conditions (B.142) and (B.151) as well as smooth-pasting

condition (B.145) at $\bar{a}$ using the simplest functional form we obtained above. We have

$$c\bar{a}^{-\gamma} = (e(a^\star)+1)\frac{\bar{a}}{a^\star} - 1, \tag{B.156}$$

$$-c\gamma\bar{a}^{-(\gamma+1)} = \frac{e(a^\star)+1}{a^\star}, \tag{B.157}$$

$$b\bar{a}^{-\gamma} = 1. \tag{B.158}$$

Other conditions at $\underline{a}$ for the equity value and the price are satisfied by construction, as well as the memoryless property condition. Note also that because $e(a) = ca^{-\gamma}$ for $a \in [0, \bar{a}]$ and $e(a)$ increases with a for $a \in [\bar{a}, \infty)$, limited liability holds for all a if $e(\bar{a}) \geq 0$, which is implied by optimization constraint (42) in the statement of the proposition.

To obtain objective function (41) from (35), we derive $\mathbb{E}[e(Sa^\star) + p(Sa^\star)]$ using (B.154) and (B.155). Setting $\varphi = 0$ and using $\underline{a} = 0$ and $\underline{e} = 0$, we obtain

$$
\begin{aligned}
\mathbb{E}[e(Sa^\star)+p(Sa^\star)] &= \frac{\xi}{\xi+1}\left[1-\left(\frac{a^\star}{\bar{a}}\right)^{-(\xi+1)}\right](e(a^\star)+1) - \left[1-\left(\frac{a^\star}{\bar{a}}\right)^{-\xi}\right]+\frac{\xi c\bar{a}^{-\gamma}}{\xi-\gamma}\left(\frac{a^\star}{\bar{a}}\right)^{-\xi} \\
&\quad + \left[1-\left(\frac{a^\star}{\bar{a}}\right)^{-\xi}\right]+\frac{\xi b\bar{a}^{-\gamma}}{\xi-\gamma}\left(\frac{a^\star}{\bar{a}}\right)^{-\xi} \\
&= \left[\frac{\xi}{\xi+1}\left(1-\left(\frac{a^\star}{\bar{a}}\right)^{-(\xi+1)}\right)+\frac{\xi}{\xi-\gamma}\left(\frac{a^\star}{\bar{a}}\right)^{-(\xi+1)}\right](e(a^\star)+1),
\end{aligned}
\tag{B.159}
$$

where to obtain the second line, we used conditions (B.156) and (B.158). Substituting (B.159) into (35), we obtain expression (41) for the platform's objective function. This expression shows that $(e(a^\star)+1)/(a^\star)$ is a function only of variables $\{\bar{a}, a^\star, \gamma\}$. Hence, equations (B.156) to (B.158) define b and c as a function of these parameters and leave one constraint for the optimization problem. Combining (B.156) and (B.157), we obtain (42). This concludes the proof.

B.12   Proof of Proposition 9

The problem of an uncollateralized platform under limited commitment is to maximize

$$\frac{e(a^\star)+1}{a^\star} = \frac{\ell(a^\star)/a^\star}{r + \frac{\lambda}{\xi+1} - \mu + \left(\frac{\lambda\xi}{\xi+1} - \frac{\lambda\xi}{\xi-\gamma}\right)\left(\frac{a^\star}{\bar{a}}\right)^{-(\xi+1)}} \tag{B.160}$$

$$\text{subject to} \qquad e(\bar{a}) \equiv \left[e(a^\star)+1\right]\frac{\bar{a}}{a^\star} - 1 = -\frac{1}{1+\gamma} > 0, \tag{B.161}$$

$$a^\star \geq \bar{a}. \tag{B.162}$$

Step 1: $\underline{\delta} = \infty$. We first show that $\lim_{\underline{\delta}\to\infty} \gamma = -\infty$. We showed that the parameter γ depends only on policy parameter $\underline{\delta}$ and that $\partial\gamma/\partial\underline{\delta} < 0$ for all $\underline{\delta}$ in Section B.10. Remember also that γ is defined as the unique root strictly lower than -1 from the characteristic polynomial (40). This polynomial can be rewritten as

$$f(x) = -\frac{\sigma^2}{2}x^3 + \left(\frac{\sigma^2}{2}\xi - \frac{\sigma^2}{2} + \mu - \underline{\delta}\right)x^2 + \left(\frac{\sigma^2}{2}\xi - (\mu - \underline{\delta})\xi + r + \lambda - \underline{\delta}\right)\left(x - r\xi + \underline{\delta}\xi\right) \tag{B.163}$$

$$= -\frac{\sigma^2}{2}(x - \gamma_1)(x - \gamma_2)(x - \gamma_3), \tag{B.164}$$

where we differentiate between the 3 roots such that $\gamma_1 \geq \gamma_2 \geq \gamma_3 = \gamma$. Matching (B.163) with (B.164), the coefficient of the second-order term is

$$\frac{\sigma^2}{2}\xi - \frac{\sigma^2}{2} + \mu - \underline{\delta} = \frac{\sigma^2}{2}(\gamma_1 + \gamma_2 + \gamma_3). \tag{B.165}$$

Since $\gamma_1 \geq \gamma_2 > -1$ and $\gamma_3 < -1$, it must be that $\lim_{\underline{\delta}\to\infty} \gamma = -\infty$. Hence, to show that $\underline{\delta} = \infty$, it is enough to show that $\gamma = -\infty$ is optimal.

Suppose by contradiction that there exists a solution with $\gamma > -\infty$. We define below an alternative set of parameters $(\tilde{a}^\star, \widetilde{\bar{a}}, \tilde{\gamma})$ that satisfy constraints (B.161) and (B.162) and dominate the candidate solution. For the alternative policy, set $\tilde{a}^\star = a^\star$, $\tilde{\gamma} = -\infty$, and define $\widetilde{\bar{a}}$ implicitly such that

$$\frac{\ell(a^\star)}{r + \frac{\lambda}{\xi+1} - \mu + \frac{\lambda\xi}{\xi+1}\left(\frac{a^\star}{\widetilde{\bar{a}}}\right)^{-(\xi+1)}}\frac{\widetilde{\bar{a}}}{a^\star} = 1. \tag{B.166}$$

83

That is, we choose $\widetilde{\overline{a}}$ such that (B.161) holds as an equality for the same value of $a^\star$ but with $\tilde{\gamma} = -\infty$.

We show now that there exists $\widetilde{\overline{a}}$ defined by (B.166) that satisfies (B.162) and such that the platform's value (B.160) is strictly higher for the alternative parameter constellation than for the candidate solution. First, we derive that inequality condition: $(\tilde{e}(a^\star)+1)/a^\star \geq (e(a^\star)+1)/a^\star$ if and only if

$$0 \leq 1 - \frac{\xi+1}{\xi-\gamma} - \left(\frac{\widetilde{\overline{a}}}{a^\star}\right)^{\xi+1}. \tag{B.167}$$

Thus, we just need to prove (B.167) and that $\widetilde{\overline{a}} \leq a^\star$ exists. To do so, below we show that $\widetilde{\overline{a}} \leq \overline{a}(1+\gamma)/\gamma$ and that

$$0 \leq 1 - \frac{\xi+1}{\xi-\gamma} - \left(\frac{1+\gamma}{\gamma}\right)^{\xi+1}. \tag{B.168}$$

Then, $\widetilde{\overline{a}} \leq a^\star$ follows from $\widetilde{\overline{a}} \leq \overline{a}(1+\gamma)/\gamma \leq \overline{a}$ as $\gamma < -1$ and $\overline{a} \leq a^\star$. Furthermore, condition (B.167) follows from inequalities (B.168) and $\widetilde{\overline{a}}/\overline{a} \leq (1+\gamma)/\gamma$ and the fact that $w \mapsto w^{\xi+1}$ is strictly increasing.

To prove $\widetilde{\overline{a}} \leq \overline{a}(1+\gamma)/\gamma$ and condition (B.168), denote $\tilde{x} = \widetilde{\overline{a}}/a^\star$ and define $F(x)$ as

$$F(x) = \frac{\ell(a^\star)x}{r + \frac{\lambda}{\xi+1} - \mu + \frac{\lambda\xi}{\xi+1}x^{\xi+1}}.$$

We can then rewrite equation (B.166) that defines $\widetilde{\overline{a}}$ as

$$F(\tilde{x}) = 1 = \frac{\ell(a^\star)\frac{1+\gamma}{\gamma}\frac{\overline{a}}{a^\star}}{r + \frac{\lambda}{\xi+1} - \mu + \left(\frac{\lambda\xi}{\xi+1} - \frac{\lambda\xi}{\xi-\gamma}\right)\left(\frac{\overline{a}}{a^\star}\right)^{\xi+1}}, \tag{B.169}$$

where the second equality follows from the fact that the candidate solution must satisfy (B.161). Note that $F(0) = 0 < 1$. Thus, following the intermediate value theorem, if $F\left(\frac{1+\gamma}{\gamma}\frac{\overline{a}}{a^\star}\right) > 1$, then $\widetilde{\overline{a}} \leq \overline{a}(1+\gamma)/\gamma$. Given that the middle term of (B.169) must be equal to 1, $F\left(\frac{1+\gamma}{\gamma}\frac{\overline{a}}{a^\star}\right) > 1$ is equivalent to

$$0 < \left(\frac{\lambda\xi}{\xi+1} - \frac{\lambda\xi}{\xi-\gamma}\right)\left(\frac{\overline{a}}{a^\star}\right)^{\xi+1} - \frac{\lambda\xi}{\xi+1}\left(\frac{1+\gamma}{\gamma}\frac{\overline{a}}{a^\star}\right)^{\xi+1}, \tag{B.170}$$

84

which is equivalent to (B.168) after multiplying all terms by $\frac{\xi+1}{\lambda\xi}\left(\frac{a^\star}{\bar{a}}\right)^{\xi+1}$.

To prove (B.168), we introduce auxiliary variable $y \equiv 1 - \gamma^{-1}$, which lies in $(0,1)$ when $\gamma \in (-\infty, -1)$. Substituting for $\gamma = -(1-y)^{-1}$ in (B.168), this condition is equivalent to

$$1 - y^{\xi+1} > 1 - \frac{y}{\xi(1-y)+1} \quad \Leftrightarrow \quad 0 \le y^{-\xi} - \xi(1-y) - 1 \equiv G(y). \quad \text{(B.171)}$$

We have $G(1) = 0$ and

$$G'(y) = \xi(1 - y^{-\xi-1}) < 0 \quad \text{(B.172)}$$

because $y < 1$ and thus $(1/y)^{\xi+1} > 1$. This proves that $G(y) > 0$ for all $0 < y < 1$. Hence, we established condition (B.168), which proves that $\underline{\delta} = \infty$ is optimal.

Step 2. Second, we prove the result about $a^\star$. Given policy choice $\underline{\delta} = \infty$ such that $\gamma = -\infty$, the maximization problem of the platform given by (41) becomes

$$\max_{\bar{a},a^\star} \frac{\ell(a^\star)/a^\star}{r + \frac{\lambda}{\xi+1} - \mu + \frac{\lambda\xi}{\xi+1}\left(\frac{a^\star}{\bar{a}}\right)^{-(\xi+1)}} \quad \text{(B.173)}$$

$$\text{subject to} \quad e(\bar{a}) \equiv \left[e(a^\star) + 1\right]\frac{\bar{a}}{a^\star} - 1 = 0. \quad \text{(B.174)}$$

This problem is similar to that under full commitment, given by (22), except that the term $\lambda\xi/(\xi+1) - \lambda\xi/(\xi-\gamma)$ in the denominator of (22) is replaced by $\lambda\xi/(\xi+1)$ in the denominator of (41). This term loads negatively on $a^\star$ and is larger in the limited commitment case, which implies that $a^\star$ should be higher under limited commitment than under full commitment.

We now prove that the necessary condition for a platform to exist is tighter under limited commitment. The optimization problem under full commitment, given by (22) and that under no commitment, given by (B.173), can be nested under the following specification:

$$\max_{\bar{a},a^\star} \frac{\ell(a^\star)/a^\star}{u + v(\gamma)\left(\frac{a^\star}{\bar{a}}\right)^{-(\xi+1)}} \quad \text{(B.175)}$$

$$\text{subject to} \quad e(\bar{a}) \equiv \left[e(a^\star) + 1\right]\frac{\bar{a}}{a^\star} - 1 = 0, \quad \text{(B.176)}$$

with

$$u = r + \frac{\lambda}{\xi+1} - \mu, \quad v(\gamma) = \frac{\lambda\xi}{\xi+1} - \frac{\lambda\xi}{\xi-\gamma}.$$

We have $\gamma = -\infty$ in the limited-commitment case, as shown above, while $\gamma > -\infty$ is the solution to (21) in the full-commitment case. We can thus use the analysis in the proof of Proposition 5, which shows that a necessary condition for existence of an uncollateralized platform is (B.90). Observe that $v(\gamma)$ is increasing with $\gamma < 0$. Hence, to show that the existence condition is tightest under limited commitment, we are left to show that the right-hand side of (B.90) increases with $v(\gamma)$. We have

$$\frac{u + v(\gamma)\min\left\{1, \frac{u}{v(\gamma)\xi}\right\}}{\min\left\{1, \frac{u}{v(\gamma)\xi}\right\}^{\frac{1}{\xi}+1}} = \begin{cases} u + v(\gamma) & \text{if } v(\gamma) \leq \frac{u}{\xi} \\ \frac{u(\xi+1)}{\xi}\left(\frac{v(\gamma)\xi}{u}\right)^{\frac{1}{\xi}+1} & \text{if } v(\gamma) \geq \frac{u}{\xi} \end{cases}. \tag{B.177}$$

The desired result follows immediately from inspection of (B.177).

B.13   Proof of Proposition 11

We derive arbitrage relationship (51) from the HJB for the vault value, $V(A, C, C^i)$. We first show that returns to issuance are zero when condition (50) holds. A vault owner solves

$$v(a)C_-^i = \max_{d\mathcal{G}^i}\left\{p(a)\mathbb{E}\left[d\mathcal{G}^i\right] - \mathbb{E}\left[d\mathcal{M}^i\right] + (1-rdt)\mathbb{E}\left[v(a+da)(C_-^i + dC^i)\right]\right\}. \tag{B.178}$$

Substituting for dC^i, we get

$$v(a)C_-^i = \max_{d\mathcal{G}^i}\left\{p(a)\mathbb{E}\left[d\mathcal{G}^i\right] - \varphi\mathbb{E}\left[s(a)C_-^i dt + d\mathcal{G}^i - \mu^k C_-^i dt\right]\right.$$
$$\left. + (1-rdt)\mathbb{E}\left[v(a+da)(C_-^i + s(a^\star)C^i dt + d\mathcal{G}^i)\right]\right\}. \tag{B.179}$$

The first-order condition for $d\mathcal{G}^i$ is given by

$$p(a) - \varphi + v(a) = 0, \tag{B.180}$$

which is equivalent to (50). Hence, a vault owner enjoys the same value irrespectively of its issuance $d\mathcal{G}^i$. We can thus rewrite the HJB for the vault value as follows

$$v(a) = \varphi(\mu^k - s(a))dt + (1-rdt)\mathbb{E}\left[v(a+da)(1 + s(a)dt)\right], \tag{B.181}$$

where we divided by the current stock C_-^i of the vault's stablecoins. Substituting for $v(a)$ using (B.180), we get

$$(\varphi - p(a)) = \varphi(\mu^k - s(a))dt + (1 - rdt)\mathbb{E}\left[(\varphi - p(a + da))(1 + s(a)dt)\right]. \qquad (B.182)$$

Expanding the expectation term on the right-hand side of (B.182) and keeping only terms of order dt, we obtain

$$0 = \varphi(\mu^k - s(a)) + (s(a) - r)(\varphi - p(a)) - \mu^p(a)p(a), \qquad (B.183)$$

which is equivalent to (51) and where $\mu^p(a) \equiv \mathbb{E}[dp(a)/(p(a)dt)]$.[38] Similarly, we can write the HJB of the price as

$$p(a) = \ell(a)p(a)dt + \delta(a)p(a)dt + (1 - rdt)\mathbb{E}\left[p(a + da)\right]. \qquad (B.184)$$

Further algebra yields

$$rp(a) = \ell(a)p(a) + \delta(a)p(a) + \mu^p(a)p(a). \qquad (B.185)$$

Using equation (B.183) and (B.185), we get

$$(s(a) - \delta(a))p(a) = \ell(a)p(a) + \varphi(\mu^k - r). \qquad (B.186)$$

The maximization problem is then given by

$$E_t = \max_{\tau,s,\delta} \mathbb{E}_t\left[\int\!\!\!\int_0^T e^{-r(s-t)}\left(\ell_s p_s C_s + (\mu^k - r)\varphi C_s\right)ds\right] \qquad (B.187)$$

subject to $\varphi - p_t \geq 0$.

We are left to characterize the policy which implements $C_t = C^\star(A)$ with $C^\star(A)$ given by (54). Vault owners take the stablecoin price as given. From arbitrage condition (51), their supply function is a step function given by

$$d\mathcal{G}^i = \begin{cases} +\infty & \text{if } s(a) < (\mu^k - r)/p(a) + r - \mu^p(a), \\ -C_{t^-}^i & \text{if } s(a) > (\mu^k - r)/p(a) + r - \mu^p(a), \end{cases} \qquad (B.188)$$

---

[38]Here, we consider a deviation $dp(a)$ of order dt: $\lim_{dt\downarrow 0}|\mu^p(a)| < \infty$. Otherwise, with a discontinuity at a, the stablecoin price would have an infinite expected loss or gain rate, violating no-arbitrage.

and it is indeterminate if $s(a) = (\mu^k - r)/p(a) + r - \mu^p(a)$. To implement target $a^\star$ and the price peg, we must have $s(a^\star) = \mu^k$. In this case, however, vault owners are indifferent about any supply level. To implement $C^\star(A)$, the platform use a fee schedule contingent on the amount of stablecoins, whereby vault owners are induced to issue (buy back) stablecoins if $C > C^\star(A)$ ($C < C^\star(A)$). Such a schedule is given by (55). In this case, the only equilibrium supply is $C_t = C^\star(A)$. In particular, we have $s(a^\star) - \delta(a^\star) = \ell(a^\star) + \mu^k - r$. This last equation combined with $s(a^\star) = \mu^k$ to maintain the peg $p(a^\star) = 1$ implies that $\delta(a^\star) = r - \ell(a^\star)$.

C   Issuance Policies with a Brownian Component

In this section, we show that considering a policy function $d\mathcal{G}_t = g_t C_t dt$ instead of a more general functional form $d\mathcal{G}_t = g_t C_t dt + \kappa_t C_t dZ_t$ is without loss of generality. We prove the case for the centralized uncollateralized protocol in the smooth region but the proof can be adapted to any case. The intuition for the results is straightforward: If fighting Brownian shocks with κ_t has any expected impact on the value of equity, it will also be taken into account in the smooth issuance decision g_t and cancel out. With a stochastic term in $d\mathcal{G}_t$ we can write the value of equity in the smooth region as

$$
\begin{aligned}
E(A_t, C_t) =& \mathbb{E}[p(A_t + dA_t, C_t + d\mathcal{G}_t)d\mathcal{G}_t] \\
& + (1 - rdt - \lambda dt)\mathbb{E}[E(A_t + dA_t, C_t + d\mathcal{G}_t)] + (1 - rdt)\lambda dt \mathbb{E}[E(SA_t, C_t)].
\end{aligned}
\tag{C.189}
$$

Using Ito's lemma and the fact that terms in $dtdt$ converge to 0 faster than terms in dt, we can get

$$
\mathbb{E}[p(A_t + dA_t, C_t + d\mathcal{G}_t)d\mathcal{G}_t] = \mathbb{E}\left[p(A_t, C_t)g_t C_t dt + \sigma A p_A(A_t, C_t)\kappa_t C_t dt + \kappa_t^2 C_t^2 p_C(A_t, C_t)dt\right] \Big(
\tag{C.190}
$$

and

$$\mathbb{E}[E(A_t + dA_t, C_t + d\mathcal{G}_t)] = \mathbb{E}[E(A_t, C_t) + \mu A E_A(A_t, C_t)dt + g_t C_t E_C(A_t, C_t)dt$$

$$+ \frac{\sigma^2}{2} A_t^2 E_{AA}(A_t, C_t)dt + \frac{\kappa_t^2}{2} C_t^2 E_{CC}(A_t, C_t)dt \qquad \text{(C.191)}$$

$$+ \sigma A_t \kappa_t C_t E_{AC}(A_t, C_t)dt] \qquad \text{(C.192)}$$

The first-order condition for g_t is still given by

$$p(A, C) + E_C(A, C) = 0 \qquad \text{(C.193)}$$

while the first-order condition for κ_t is given by

$$\sigma A p_A(A, C) + \kappa C p_C(A, C) + \kappa C E_{CC}(A, C) + \sigma A E_{AC}(A, C) = 0. \qquad \text{(C.194)}$$

As

$$p_A(A, C) + E_{AC}(A, C) = 0 \qquad \text{(C.195)}$$

and

$$p_C(A, C) + E_{CC}(A, C) = 0, \qquad \text{(C.196)}$$

the first-order condition for κ_t is satisfied if and only if the first-order condition for g_t is satisfied. The HJB for $p(A, C)$ becomes

$$(r + \lambda - \delta(A, C))p(A, C) = \mu A p_A(A, C) + (g(A, C) + \delta(A, C))C p_C(A, C)$$

$$+ \frac{\sigma^2}{2} A^2 p_{AA}(A, C) + \frac{\kappa^2}{2} C^2 p_{CC}(A, C) \qquad \text{(C.197)}$$

$$+ \sigma A \kappa C p_{AC}(A, C) + \lambda \mathbb{E}[p(SA, C)]. \qquad \text{(C.198)}$$

Given that $p(A/C) = p(A, C)$, we get

$$(r + \lambda - \delta(a))p(a) = \ell(a) + \mu a p'(a) - (g(a) + \delta(a))ap'(a)$$

$$+ \frac{\sigma^2}{2}a^2 p''(a) + \frac{\kappa(a)^2}{2}(p''(a)a^2 + 2p'(a)a) \qquad \text{(C.199)}$$

$$- \sigma\kappa(a)(p'(a)a^2 + p'(a)a) + \lambda\mathbb{E}[p(Sa)]. \qquad \text{(C.200)}$$

Similarly,

$$e(a) = -\delta(a)p(a) + \mu a e'(a) + \frac{\sigma^2}{2}a^2 e''(a) + \lambda\mathbb{E}[e(Sa)] \qquad \text{(C.201)}$$

and

$$e'(a) = -\delta'(a)p(a) - \delta(a)p'(a) + \mu a e''(a) + \mu e'(a) + \frac{\sigma^2}{2}a^2 e'''(a) + \sigma^2 a e''(a) + \lambda\mathbb{E}[e'(Sa)].$$

$$\text{(C.202)}$$

Using the first-order condition for $g(a)$ and its derivatives,

$$p(a) = -e(a) + e'(a)a, \qquad \text{(C.203)}$$

$$p'(a) = e''(a)a, \qquad \text{(C.204)}$$

$$p''(a) = e'''(a)a + e''(a), \qquad \text{(C.205)}$$

we get

$$0 = (r + \lambda)(p(a) + e(a) - e'(a)a), \tag{C.206}$$

$$= \ell(a) + \delta(a)p(a) - (g(a) + \delta(a))ap'(a) + \mu a p'(a) + \frac{\sigma^2}{2}a^2 p''(a)$$

$$+ \frac{\kappa(a)^2}{2}(p''(a)a^2 + 2p'(a)a) - \sigma\kappa(p'(a)a^2 + p'(a)a) + \lambda\mathbb{E}[p(Sa)]$$

$$- \delta(a)p(a) + \mu a e'(a) + \frac{\sigma^2}{2}a^2 e''(a) + \lambda\mathbb{E}[e(Sa)]$$

$$+ \delta'(a)ap(a) + \delta(a)p'(a)a - \mu a^2 e''(a) - \mu a e'(a) - \frac{\sigma^2}{2}a^3 e'''(a) - \sigma^2 a^2 e''(a)$$

$$- \lambda\mathbb{E}[e'(Sa)a] \tag{C.207}$$

$$= \ell(a) + \delta'(a)ap(a) - g(a)ap'(a) + \kappa(a)^2/2(p''(a)a^2 + 2p'(a)a)$$

$$- \sigma\kappa(a)(p'(a)a^2 + p'(a)a). \tag{C.208}$$

Thus, in the smooth part of the equilibrium, it must be that

$$g(a) = \frac{\ell(a) + \delta'(a)ap(a) + \kappa(a)^2/2(p''(a)a^2 + 2p'(a)a) - \sigma\kappa(a)(p'(a)a^2 + p'(a)a)}{ap'(a)}. \tag{C.209}$$

Therefore, the HJB for $p(a)$ is given by

$$(r + \lambda)p(a) = \delta(a)p(a) - \delta'(a)ap(a) + \mu a p'(a) + \frac{\sigma^2}{2}a^2 p''(a) + \lambda\mathbb{E}[p(Sa)] \tag{C.210}$$

and none of the equilibrium price functions are affectec by $\kappa(a)$.

C.1 No Commitment

In the main text, we assume that a centralized platform has some commitment power with respect to the interest rate policy and the collateralization rule. As argued in Section 4, we show that the platform has no value if it cannot commit at all.

Lemma 9. *Without commitment, there is no MPE with striclty positive equity value* $E(A, C, K) > 0$ *and stablecoin price* $p(A, C, K) > 0$.

The problem of a platform without any commitment to policies is similar to that of a

91

firm that can choose whether or not to make coupon payments on perpetuity debt without defaulting. Once stablecoins/debt are issued, the firm strictly prefers not to make coupon payments because it has already captured any benefits from issuance. As a result, the platform would always set the interest payment to 0 ex post, which means that stablecoins have no value ex ante because the peg is not guaranteed. Lemma 9 thus shows that some commitment to a coupon policy is necessary; otherwise, the platform and the stablecoin it issues have no value.

Proof of Lemma 9. Note that we have

$$dC_t = \delta_t C_t dt + G_t dt + (\mathcal{G}_t - \mathcal{G}_{t^-}) \tag{C.211}$$

and

$$dK_t = \mu K_t dt + \sigma K_t dZ_t + M_t dt + K_{t^-}(S_t - 1)dN_t + (\mathcal{M}_t - \mathcal{M}_{t^-}). \tag{C.212}$$

If $\mathcal{G}_t = \mathcal{G}_{t^-}$ and $\mathcal{M}_t = \mathcal{M}_{t^-}$[39], using Ito's lemma we get

$$(r + \lambda)E(A_t, C_{t^-}, K_{t^-}) = p(A_t, C_t, K_t)G_t - M_t + \mu A_t E_A(A_t, C_t, K_t) \tag{C.214}$$

$$+ (G_t + \delta_t C_t)E_C(A_t, C_t, K_t) + (M_t + \mu K_t)E_K(A_t, C_t, K_t) \tag{C.215}$$

$$+ \frac{\sigma^2}{2}A_t^2 E_{AA}(A_t, C_t, K_t) + \frac{\sigma^2}{2}K_t^2 E_{KK}(A_t, C_t, K_t) + \sigma^2 A_t K_t E_{AK}(A_t, C_t, K_t) \tag{C.216}$$

$$+ \lambda \mathbb{E}[E(SA_t, C_t, SK_t)]. \tag{C.217}$$

Therefore, if $E_C(A, C, K)$ is strictly negative, given a strategy $\delta(A, C)$, there is always an optimal deviation to a lower interest payment $\delta(A, C) - \Delta$ where $\Delta > 0$ until $\delta(A, C) = 0$. By Proposition I of DeMarzo and He (2021), $E(A, C, K)$ is strictly decreasing in C when $p(A, C, K) > 0$.

Similarly, without commitment to $K(A, C) = \varphi C$, it is always optimal to put no collateral in the platform as $\mu^k < r$. □

---

[39]Otherwise, we get

$$E(A_t, C_{t^-}, K_{t^-}) = E(A_t, C_{t^-} + \mathcal{G}_t - \mathcal{G}_{t^-}, K_{t^-} + \mathcal{M}_t - \mathcal{M}_{t^-}) + p(A_t, C_{t^-} + \mathcal{G}_t - \mathcal{G}_{t^-}, K_{t^-} + \mathcal{M}_t - \mathcal{M}_{t^-})(\mathcal{G}_t - \mathcal{G}_{t^-}) - (\mathcal{M}_t - \mathcal{M}_{t^-}), \tag{C.213}$$

which is not impacted by δ_t.

Internet Appendix

In this appendix, we describe the algorithm to solve the full-commitment problem with collateral. We solve for $f^\star(\lambda, \varphi, a^\star) \equiv (e^\star + p^\star)/a^\star$ for $\{\lambda, \varphi, a^\star\} \in [0, 1] \times [0, 1] \times [1, 4]$ on a $40 \times 20 \times 20$ grid following the pseudo-algorithm below. Because the ODE is stiff otherwise, we constraint $g(a)$ to be greater or equal to -10. We use the Matlab function ode23.

Start with $\underline{a}_0^d = 0$, $\underline{a}_0^u = 1$, $\bar{a}_0 = 1$, $a_0^\star = 1.5$, $\mathbb{E}[p(Sa)]_0 = 1$, $i = 0$, $j = 0$, $k = 0$.

1. Define $\underline{a}_i = (\underline{a}_i^d + \underline{a}_i^u)/2$.

2. Solve for the second order ODE for $p(a)$ on $[\underline{a}_i, \bar{a}_i]$ given in Lemma 1 with $p(\underline{a}_i) = \varphi$ and $p'(\underline{a}_i) = 1e - 6$.

3. If $p(\bar{a}_i) < 1$, set $\underline{a}_{i+1}^u = \underline{a}_i$ and $\underline{a}_{i+1}^d = \underline{a}_i^d$. Otherwise, $\underline{a}_{i+1}^u = \underline{a}_i^u$ and $\underline{a}_{i+1}^d = \underline{a}_i$.

4. If $\underline{a}_{i+1}^u - \underline{a}_{i+1}^d < 1e - 6$, continue to the next step; otherwise, set $i = i + 1$ and go to step 1.

5. Solve for $\mathbb{E}[p(Sa)]_{j+1}$ given the new solution for $p(a)$.

6. If $||\mathbb{E}[p(Sa)]_{j+1} - \mathbb{E}[p(Sa)]_j|| < 1e - 5$, continue to the next step; otherwise set $j = j + 1$ and go to step 1.

7. Solve for $\bar{a}_{k+1}$ such that $e^\star(\underline{a}_i, \bar{a}_{k+1}, a^\star) = 0$.

8. If $|e^\star(\underline{a}_i, \bar{a}_{k+1}, a^\star) - e^\star(\underline{a}_i, \bar{a}_k, a^\star)| < 1e - 4$, end; otherwise, set $k = k + 1$ and go to step 1.